A short course on

Structural Reliability

This lecture:

The Basic Reliability Problem

Terje's Toolbox is freely available at terje.civil.ubc.ca
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Design Equations

$$\phi \cdot R_n = \gamma \cdot S_n$$

$$\phi \cdot M_{R,n} = \gamma_{D,n} \cdot M_{D,n} + \gamma_{L,n} \cdot M_{L,n}$$

Case	Load Combination[1]	
	Principal Loads	Companion Loads
1	1.4**D**[2]	—
2	(1.25**D**[3] or 0.9**D**[4]) + 1.5**L**[5]	1.0**S**[6] or 0.4**W**
3	(1.25**D**[3] or 0.9**D**[4]) + 1.5**S**	1.0**L**[6][7] or 0.4**W**
4	(1.25**D**[3] or 0.9**D**[4]) + 1.4**W**	0.5**L**[7] or 0.5**S**
5	1.0**D**[4] + 1.0**E**[8]	0.5**L**[6][7] + 0.25**S**[6]

Limit-state Functions

$$g(\mathbf{x}) < 0: \text{ Failure}$$
$$g(\mathbf{x}) > 0: \text{ Safe}$$
$$g(\mathbf{x}) = 0: \text{ The limit-state surface}$$

$$\phi \cdot R_n = \gamma \cdot S_n$$

$$g = R - S$$

$$g = R - S \quad \Leftrightarrow \quad g = 1 - \frac{S}{R} \quad \Leftrightarrow \quad g = \ln\left(\frac{R}{S}\right)$$

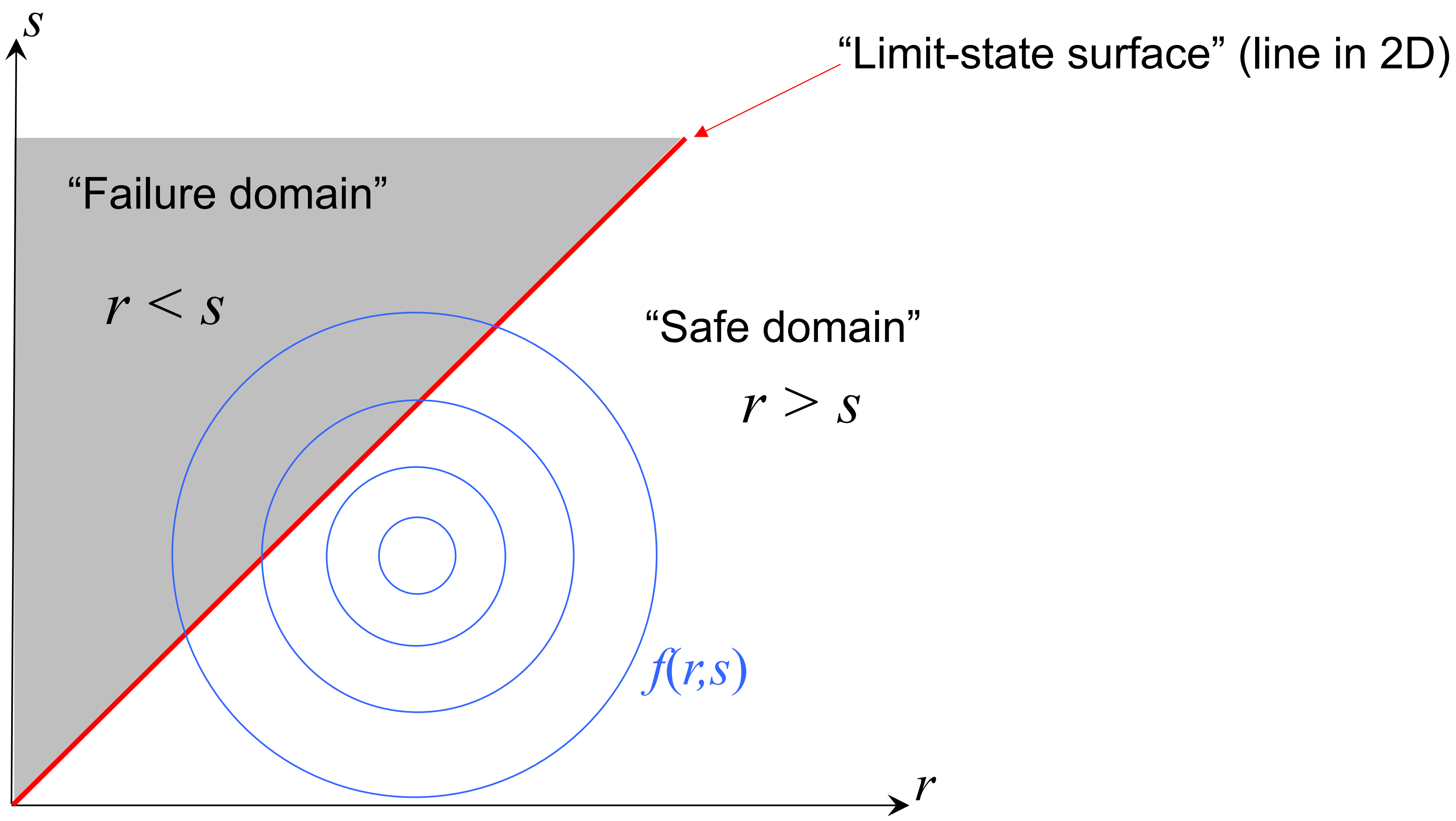
$$\phi \cdot M_{R,n} = \gamma_{D,n} \cdot M_{D,n} + \gamma_{L,n} \cdot M_{L,n}$$

$$g = M_R - M_D - M_L$$

$$g = u_o - u(\mathbf{x})$$

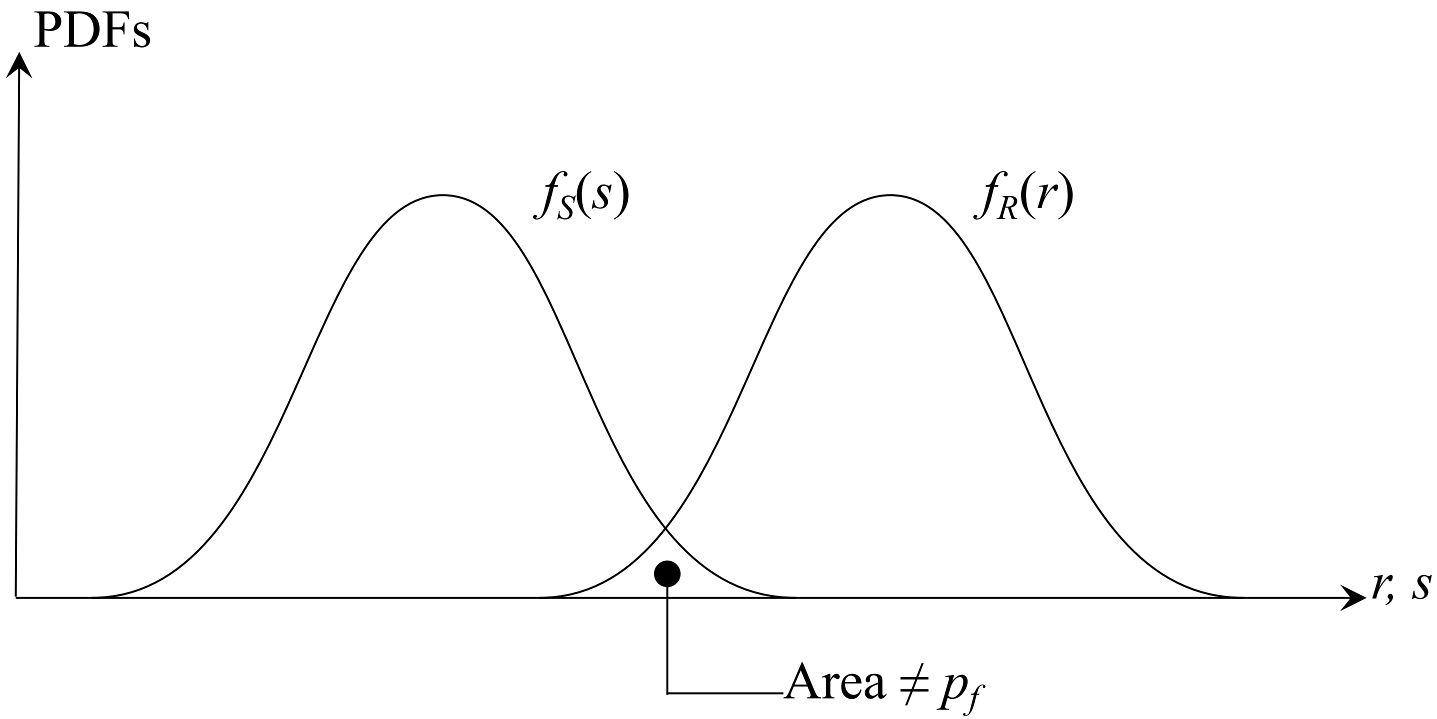
Basic Problem

$$g = R - S$$



One Approach

$$p_f = \iint_{R<S} f_R(r) f_S(s)\, dr\, ds = \int_0^{\infty}\int_0^{s} f_R(r) f_S(s)\, dr\, ds = \int_0^{\infty} F_R(s) f_S(s)\, ds$$



Another Approach

Consider: $g = R - S$

C. A. Cornell, 1969 ACI paper: $p_f = \mathrm{P}(g \le 0) = \Phi\left(\frac{g - \mu_g}{\sigma_g}\right) = \Phi\left(-\frac{\mu_g}{\sigma_g}\right) = \Phi(-\beta)$

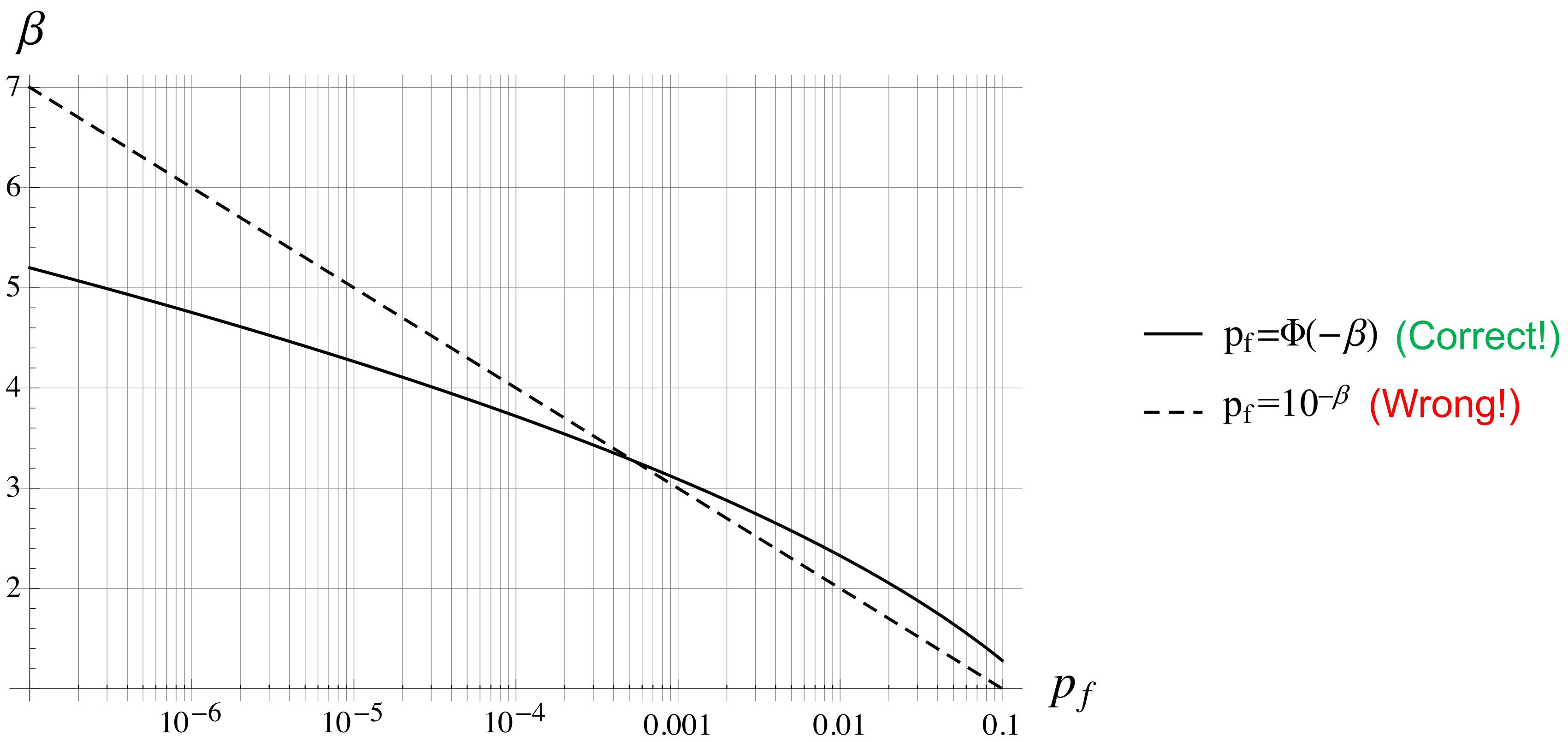
Reliability index: $\beta \equiv \frac{\mu_g}{\sigma_g} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$

$\mu_g = \mu_R - \mu_S$

$\sigma_g^2 = \nabla g^T \boldsymbol{\Sigma}_{RS} \nabla g = \sigma_R^2 + \sigma_S^2$

Requires knowledge of functions of random variables & probability transformations

β versus p_f



Invariance Problem

μ_R=30, μ_S=20, σ_R=5, σ_S=10, ρ_{RS}=0.5

Equivalent limit-state functions:

Linear: $g = R - S$

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 - 2\rho_{RS}\sigma_R\sigma_S + \sigma_S^2}} = 1.15 \text{ (Correct!)}$$

Nonlinear: $g = \ln\left(\frac{R}{S}\right)$

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{\ln(\mu_R/\mu_S)}{\sqrt{\sigma_R^2/\mu_R^2 - (2\rho_{RS}\sigma_R\sigma_S)/(\mu_R\mu_S) + \sigma_S^2/\mu_S^2}} = 0.92 \text{ (Wrong!)}$$

Requires knowledge of functions of random variables

Real Problem?

Just use $g = R - S$ ...?

No! Reliability analysis is so much more:

$$g = u_o - \frac{F \cdot L^3}{3 \cdot E \cdot I}$$ Nonlinear!

$$g = u_o - u(\mathbf{x})$$ Nonlinear, even for linear static analysis!

↑ Finite element response

Understand & Solve Invariance Problem

Functions of random variables

Probability transformations

More lectures:

Terje’s Toobox:

terje.civil.ubc.ca