A short course on

Nonlinear Finite Element Analysis

This video:

Material Nonlinearity

Terje's Toolbox is freely available at terje.civil.ubc.ca
It is created and maintained by Professor Terje Haukaas, Ph.D., P.Eng.,
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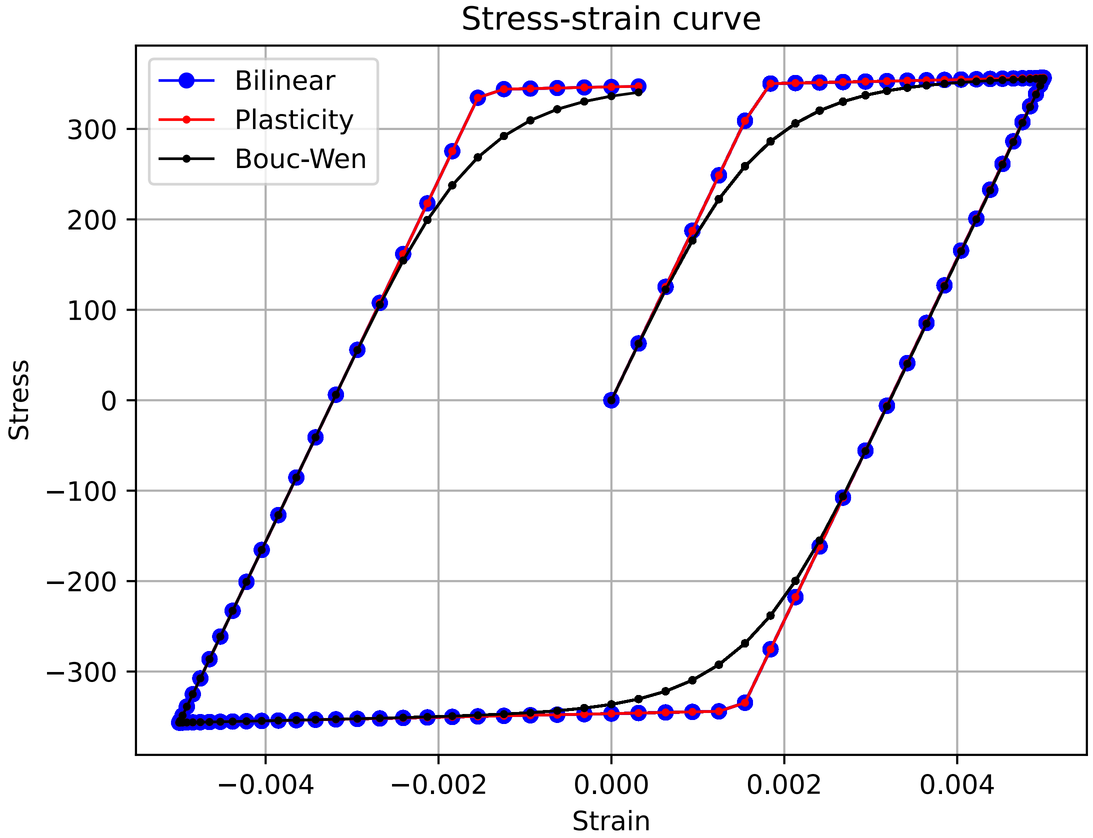
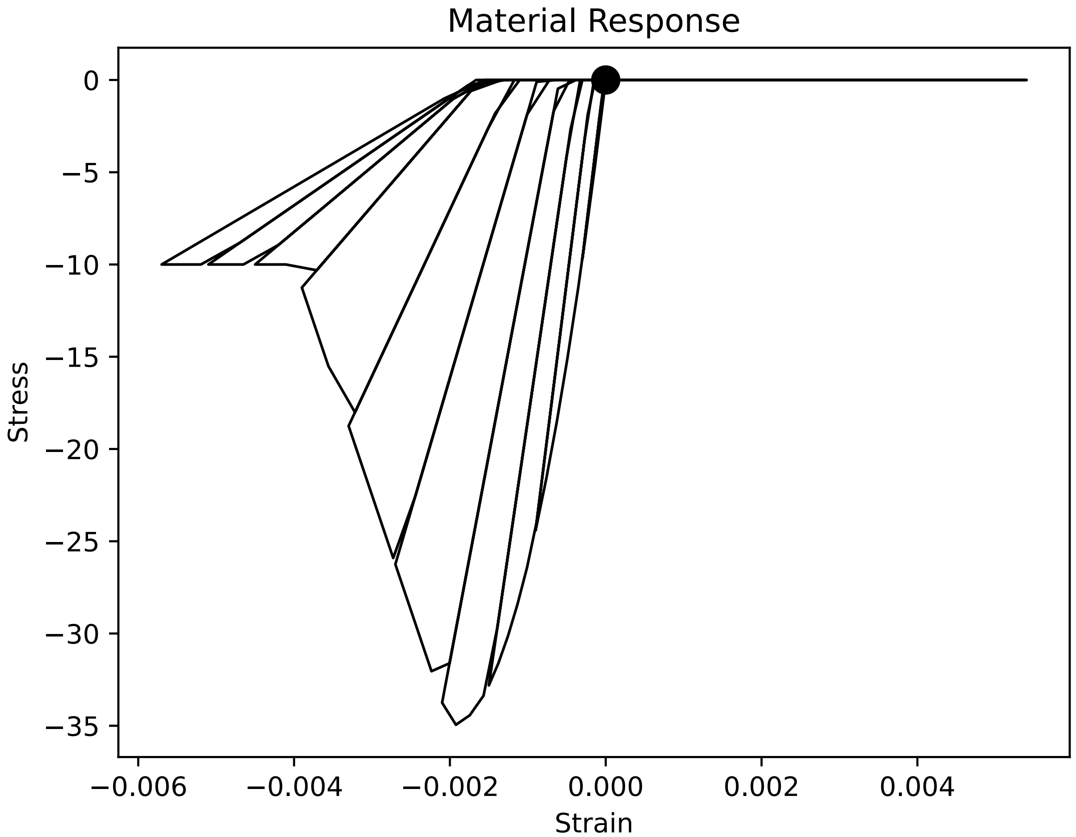
Steel

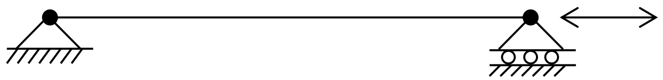

Stress-strain curve
Bilinear
Plasticity
Bouc-Wen
Stress
Strain
300
200
100
0
−100
−200
−300
−0.004
−0.002
0.000
0.002
0.004

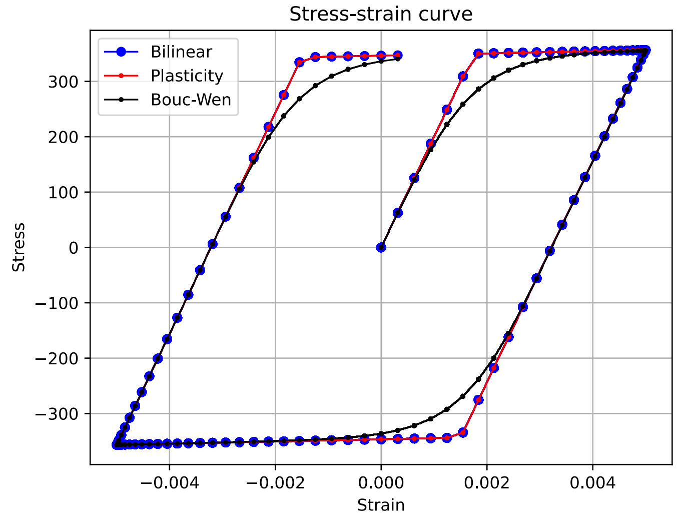
Concrete

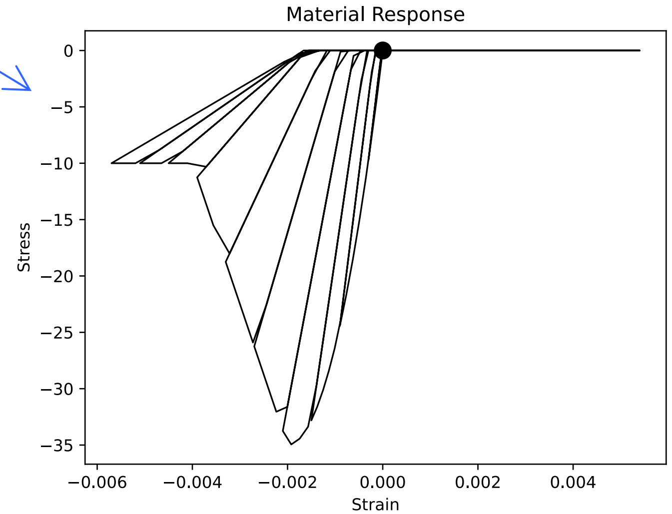


Material Response
0
−5
−10
−15
−20
−25
−30
−35
Stress
−0.006
−0.004
−0.002
0.000
0.002
0.004
Strain

Material Testers

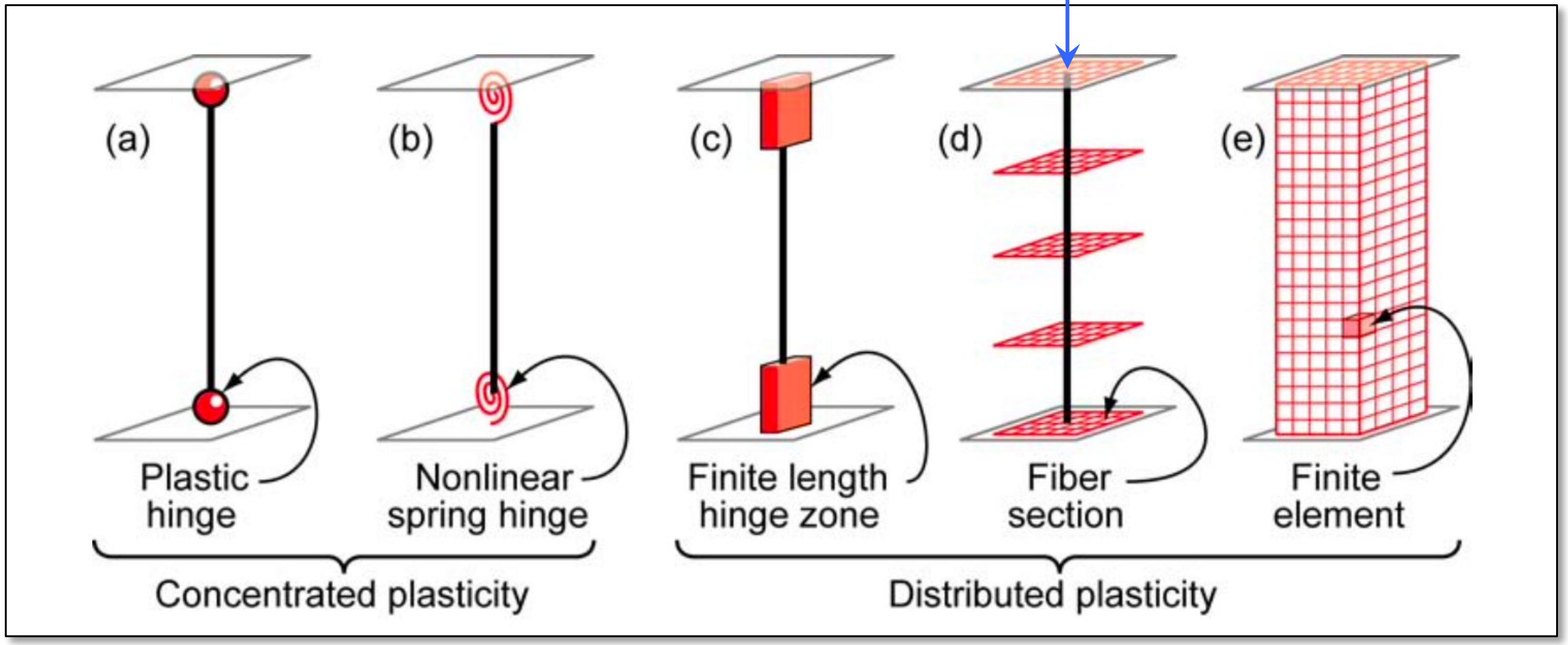
- Several “testers” posted online
- For both **G2** and **OpenSees**
- Option 1: Input strain to “state” and get stress
- Option 2: Test implementation too: 





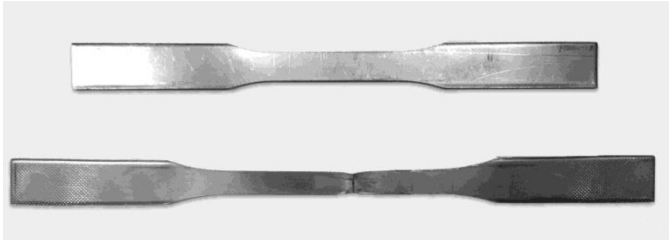
Uniaxial Material Models

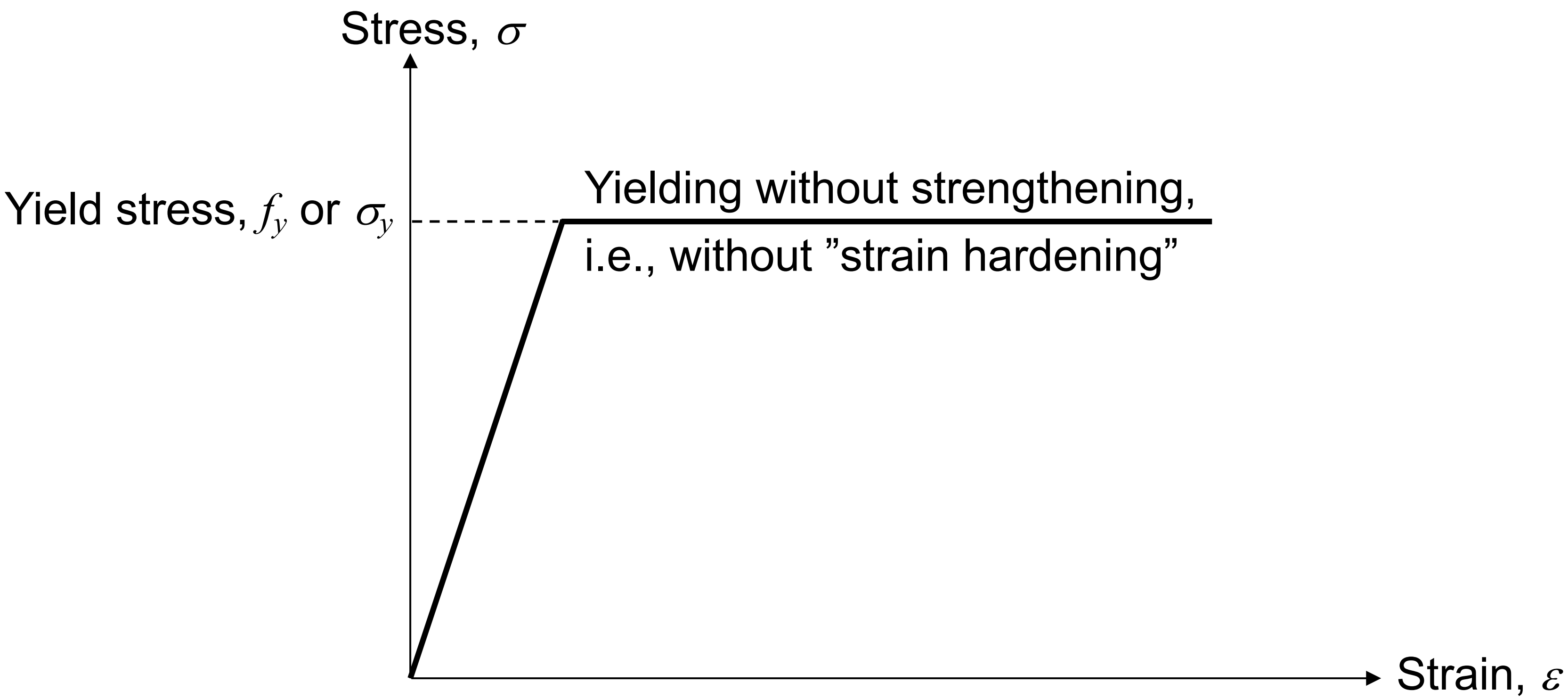
Used in fibres in distributed plasticity elements
(displacement-based and force-based)



NIST GCR 10-917-5, NEHRP Seismic Design Technical Brief No. 4, Nonlinear Structural Analysis For Seismic Design, A Guide for Practicing Engineers, by Gregory G. Deierlein, Andrei M. Reinhorn, and Michael R. Willford.

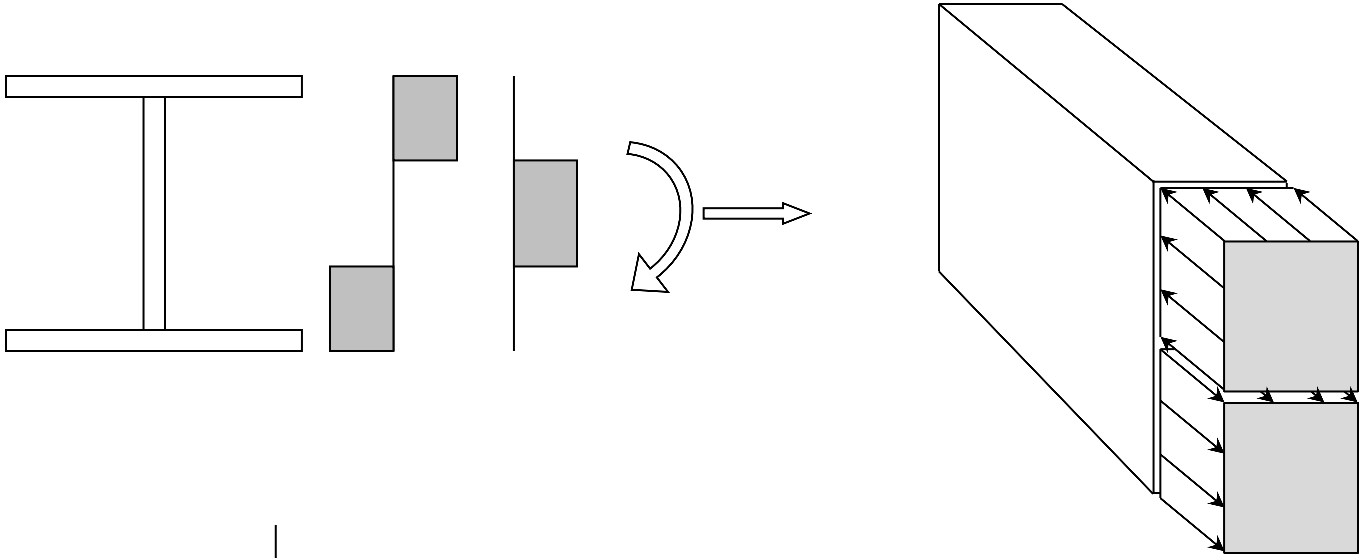
Elasto-Plastic



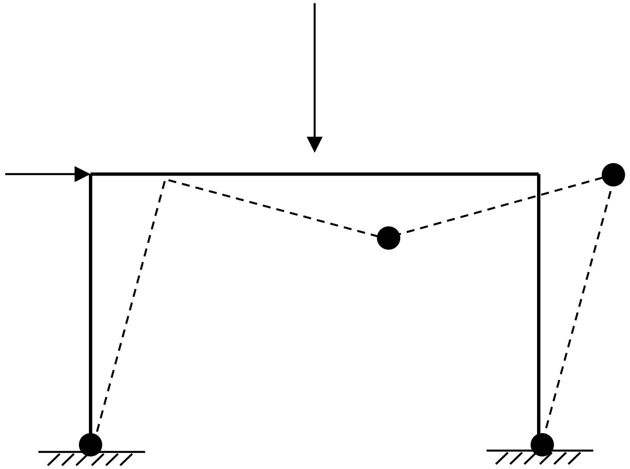


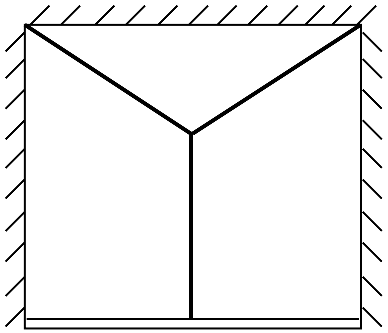
Plastic Capacity Analysis

- Lower-bound theorem (equilibrium)



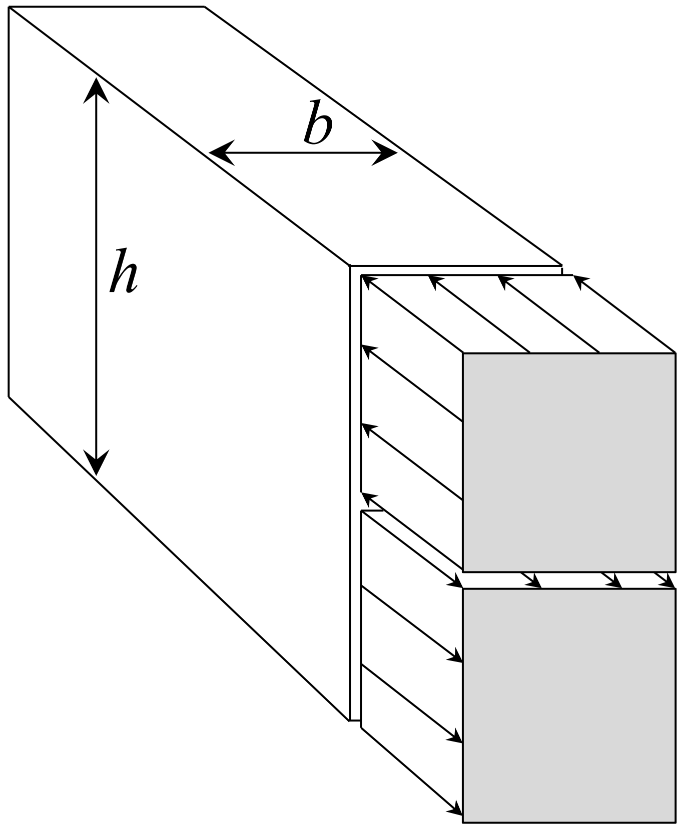
- Upper-bound theorem (compatibility)





Lower-bound

$$M_u = \sigma_y \cdot \left(b \cdot \frac{h}{2}\right) \cdot \frac{h}{2}$$

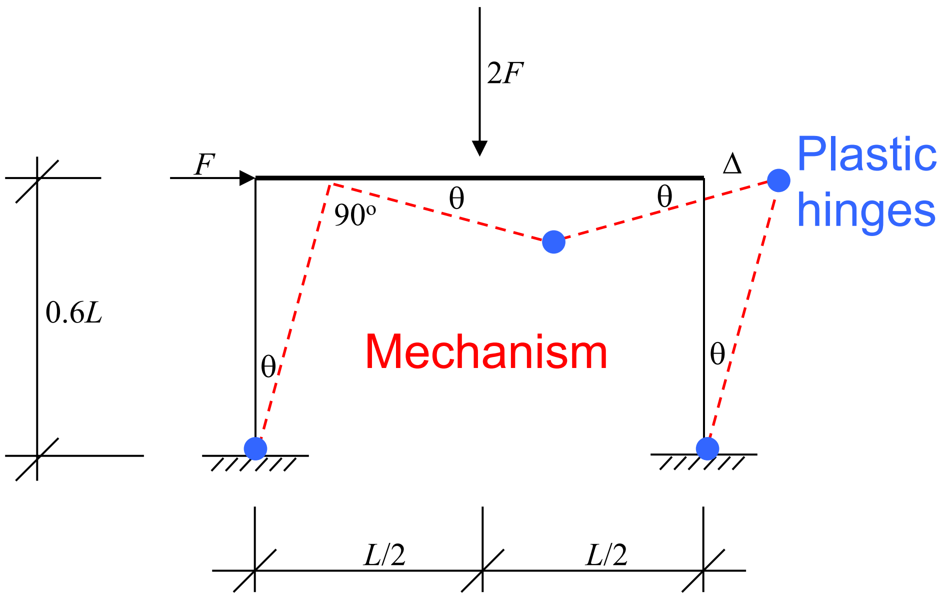


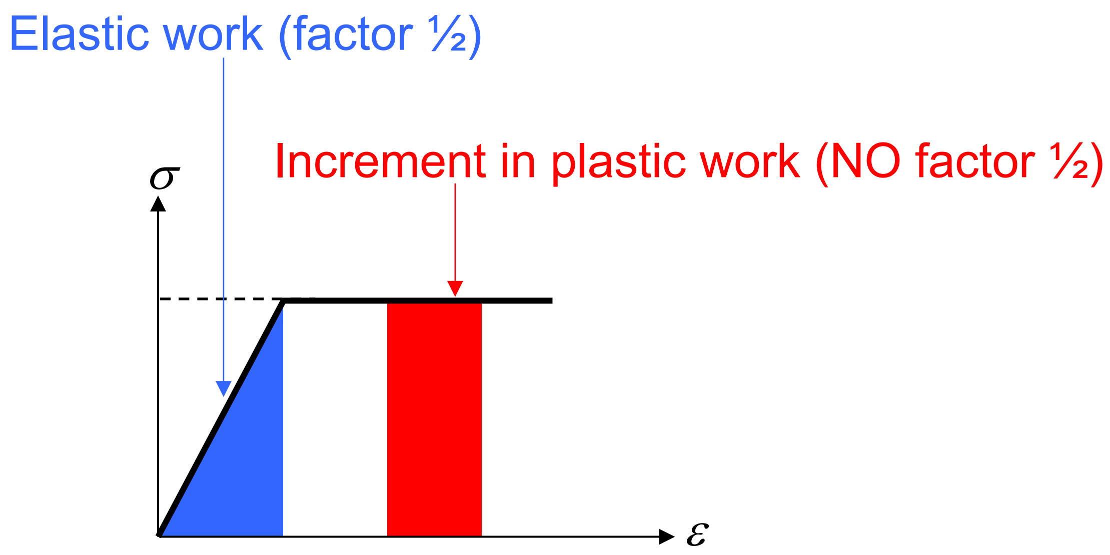
Upper-bound

$$dW_{int} = (2\theta + \theta + \theta) \cdot M_u + 2\theta \cdot 2M_u$$

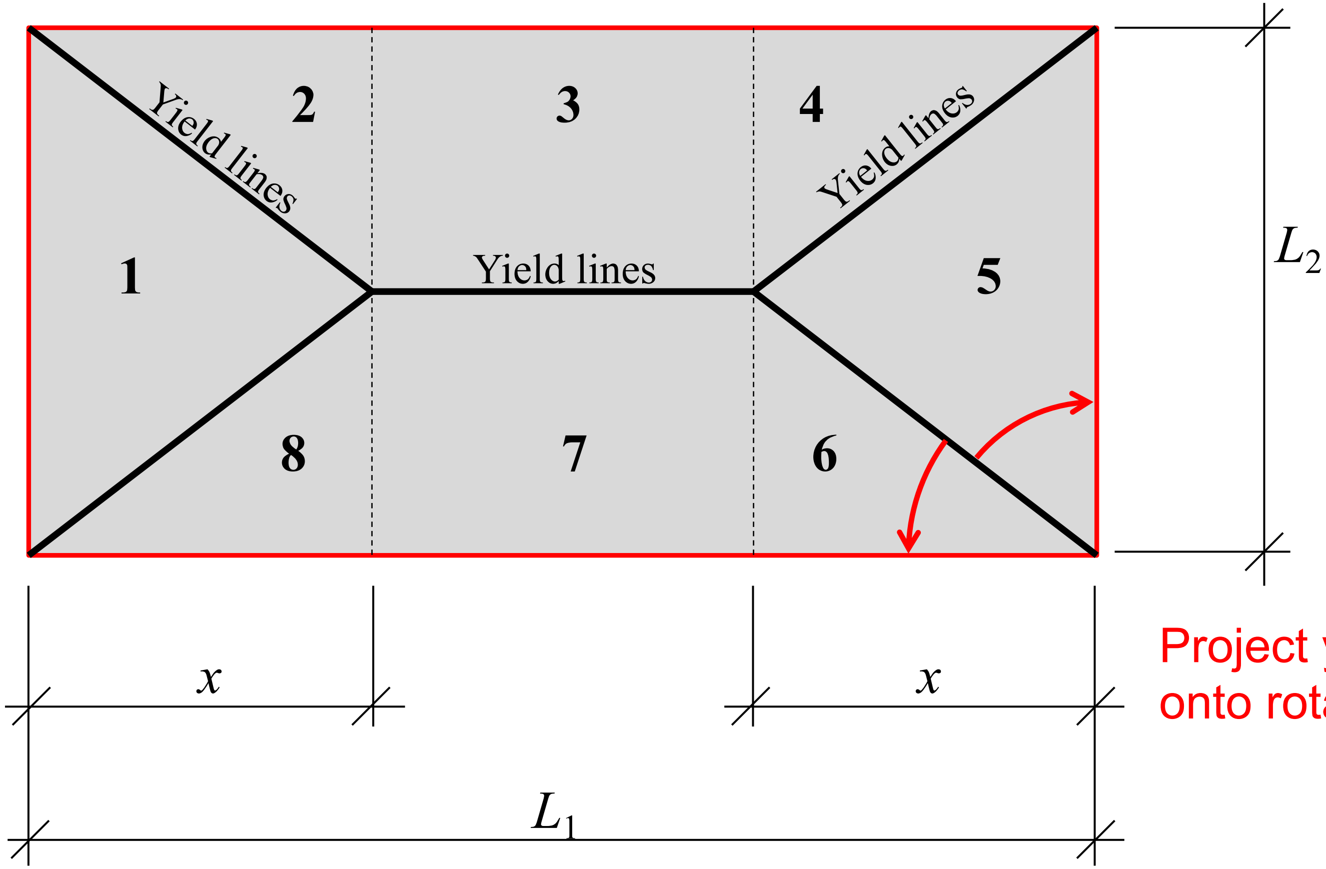
$$dW_{ext} = P(0.6L \cdot \theta) + 2P\left(\frac{L}{2} \cdot \theta\right)$$

$$dW_{int} = dW_{ext} \Rightarrow F_u = \frac{5M_u}{L}$$





Plates



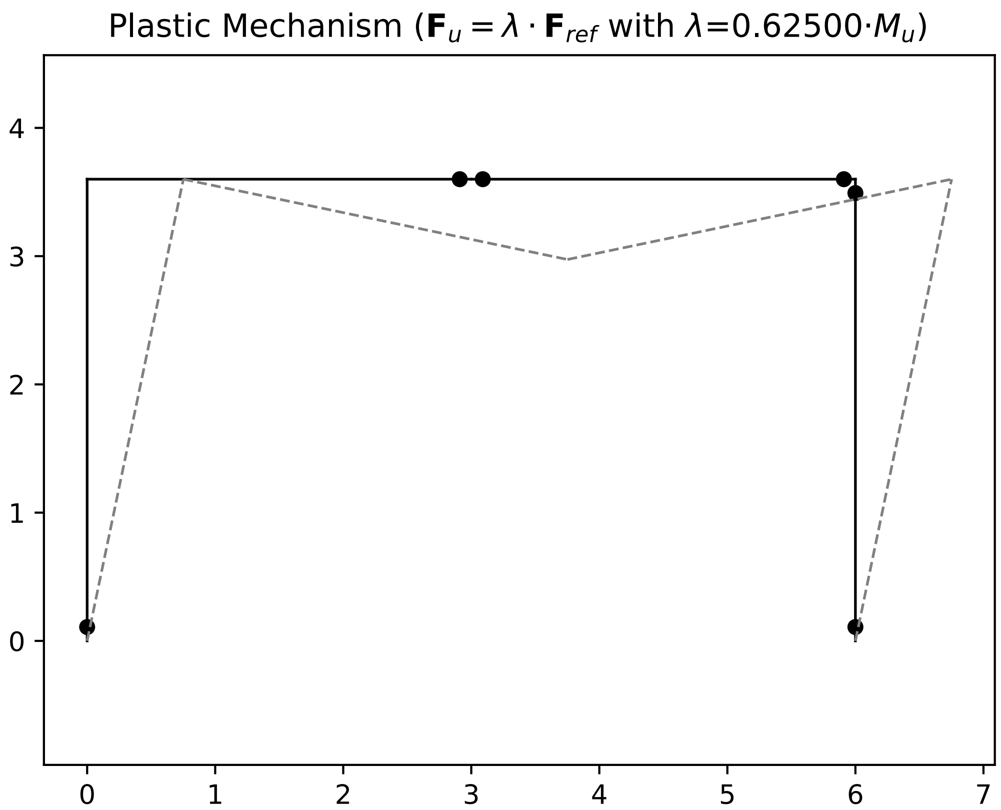
Project yield lines
onto rotating edges

Pros and Cons

- Quick estimates of “ultimate capacity” by hand calculations
- Can also be done computationally
- Concept employed in “capacity design” procedures
- Large deformations may develop before capacity is reached
- The upper-bound theorem is unconservative; must try different mechanisms

Python

See code posted at terje.civil.ubc.ca



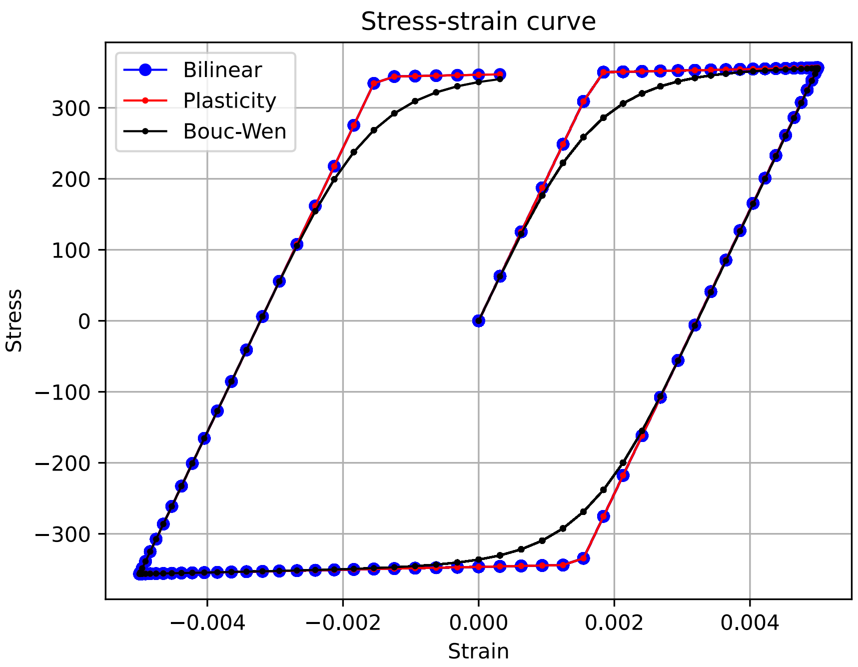
Hysteresis

- Input variables
- State variables
- History variables
- Incremental strain
- Commit!

```
# increment     = current load step, i.e., increment
# dlamb         = load increment
# theLambda     = load factor for current load step
# iter          = counter of equilibrium iterations
# ua            = matrix of displacements for "all" DOFs
#                 First column of ua  = total displacements
#                 Second column of ua = displacements since last committed state
#                 Third column of ua  = solution to system of equations
# ua1old        = previously committed displacements (used only if solution does not converge)
```

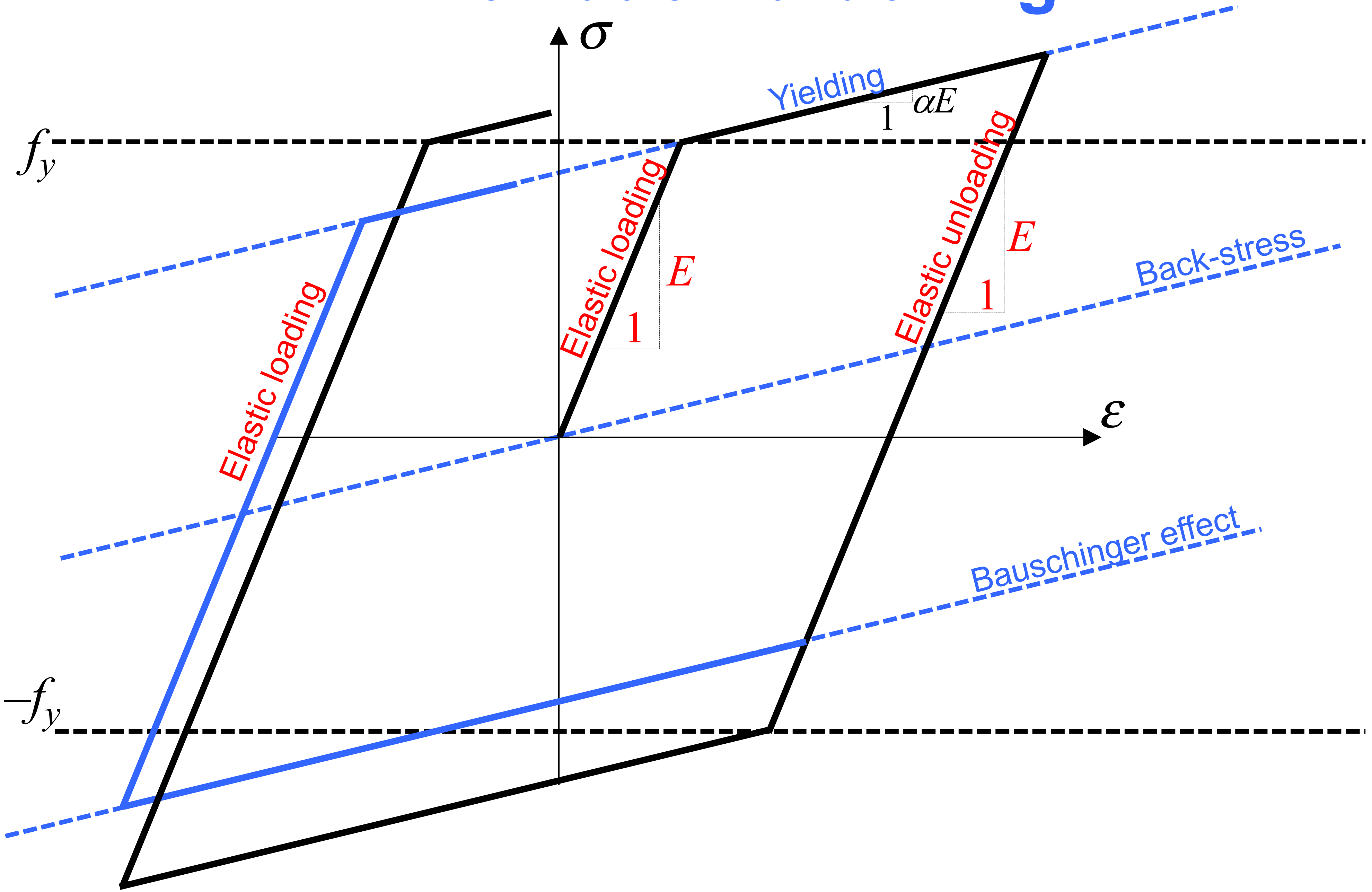
```python
class bilinearMaterial():

    # ------------------------------------------------
    # Constructor
    # ------------------------------------------------
    def __init__(self, mat):

        # Material and geometry properties
        self.E = float(mat[1])       # Modulus of elasticity
        self.fy = float(mat[2])      # Yield stress
        self.alpha = float(mat[3])  # Second slope stiffness is alpha*E

        # Trial instantiation variables
        self.trialStrain = 0.0
        self.trialStress = 0.0
        self.trialBackStress = 0.0
        self.trialYielding = False

        # Committed instantiation variables
        self.committedStrain = 0.0
        self.committedStress = 0.0
        self.committedBackStress = 0.0
        self.committedYielding = False

```

Bilinear Model



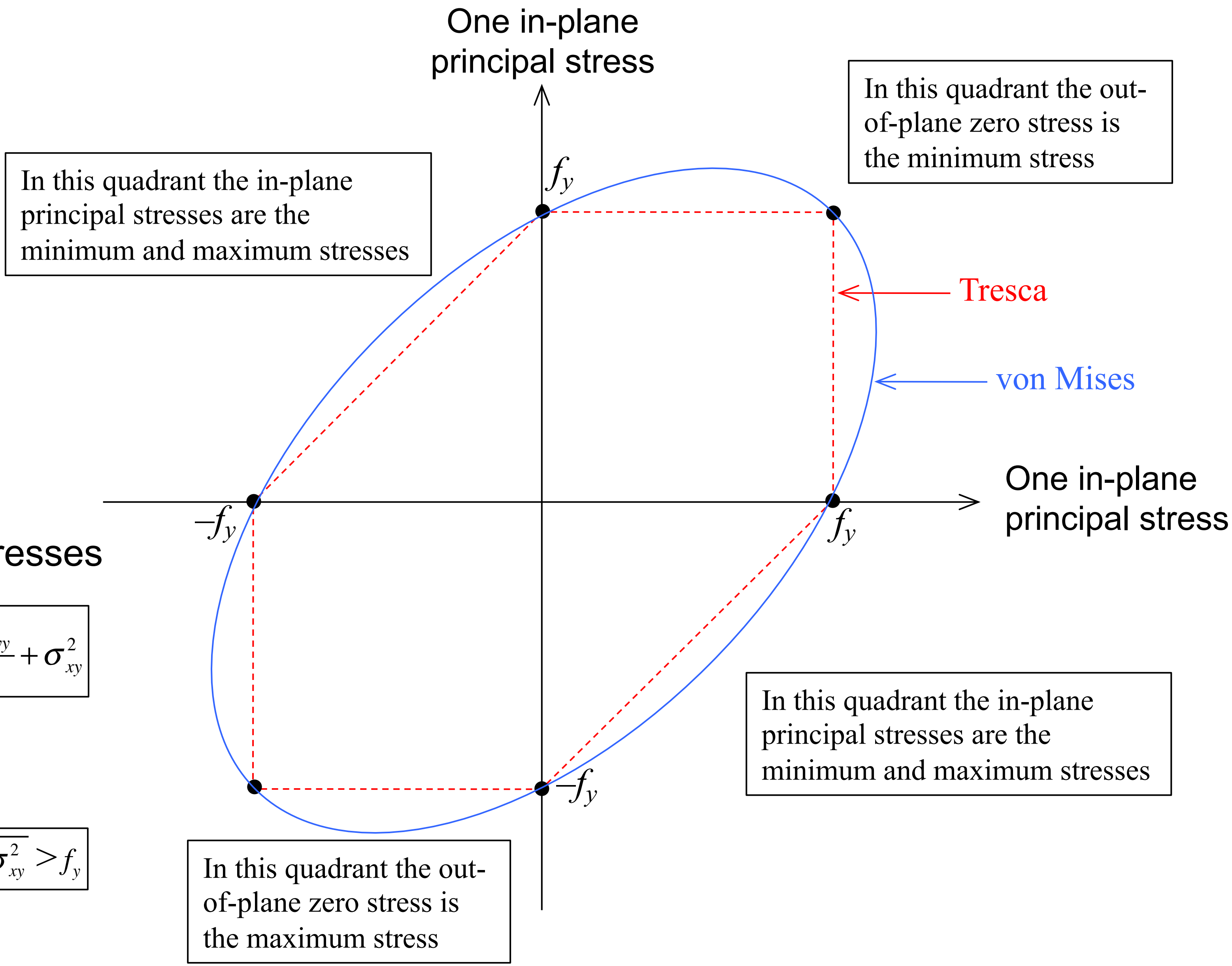

```python
def state(self, eps):

    # This is an incremental material
    deps = eps[1]

    # Check for unloading
    unloading = False
    if (self.committedYielding == True and (self.committedStress - self.committedBackStress) * deps < 0):
        unloading = True

    # Check if the last state was elastic or if the strain increment implies unloading from yielding
    if self.committedYielding is False or unloading:

        # Strain increment that would cause yielding
        depsToYield = (np.sign(deps) * self.fy + self.committedBackStress - self.committedStress) / self.E

        # Keep elastic state handy, in case that becomes the conclusion
        self.trialStress = self.committedStress + self.E * deps
        Et = self.E
        self.trialYielding = False

        # Check if the strain increment causes yielding from an elastic state (initiation of unloading is elastic)
        if abs(deps) > abs(depsToYield) and not unloading:
            self.trialStress = self.trialStress + (self.alpha * self.E - self.E) * (deps - depsToYield)
            self.trialBackStress = self.committedBackStress + self.alpha * self.E * (deps - depsToYield)
            Et = self.alpha * self.E
            self.trialYielding = True

    else:

        # Continue plastic loading
        self.trialStress = self.committedStress + self.alpha * self.E * deps
        self.trialBackStress = self.committedBackStress + self.alpha * self.E * deps
        Et = self.alpha * self.E
        self.trialYielding = True

    return self.trialStress, Et
```

Kinematic Hardening



Yielding Criteria

- Tresca: "It is shear stress"
- Mohr: $\tau_{max}=½(\sigma_{max}-\sigma_{min})$
- Tresca: Yielding if $(\sigma_{max}-\sigma_{min}) > f_y$
- von Mises:
  - Deviatoric stress tensor
  - Eigenvalue problem for principal stresses
  - Use J_2: $J_2 = \frac{\sigma^2}{3}$ $\quad J_2 = \frac{\sigma_{xx}^2}{3} + \frac{\sigma_{yy}^2}{3} - \frac{\sigma_{xx}\sigma_{yy}}{3} + \sigma_{xy}^2$
  - Yielding if: $\sqrt{3 \cdot J_2} > f_y$
  - Result: $\sigma > f_y$ $\quad \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 - \sigma_{xx}\sigma_{yy} + 3 \cdot \sigma_{xy}^2} > f_y$



J_2 Plasticity

- Yield function: $f = |\sigma| - f_y$

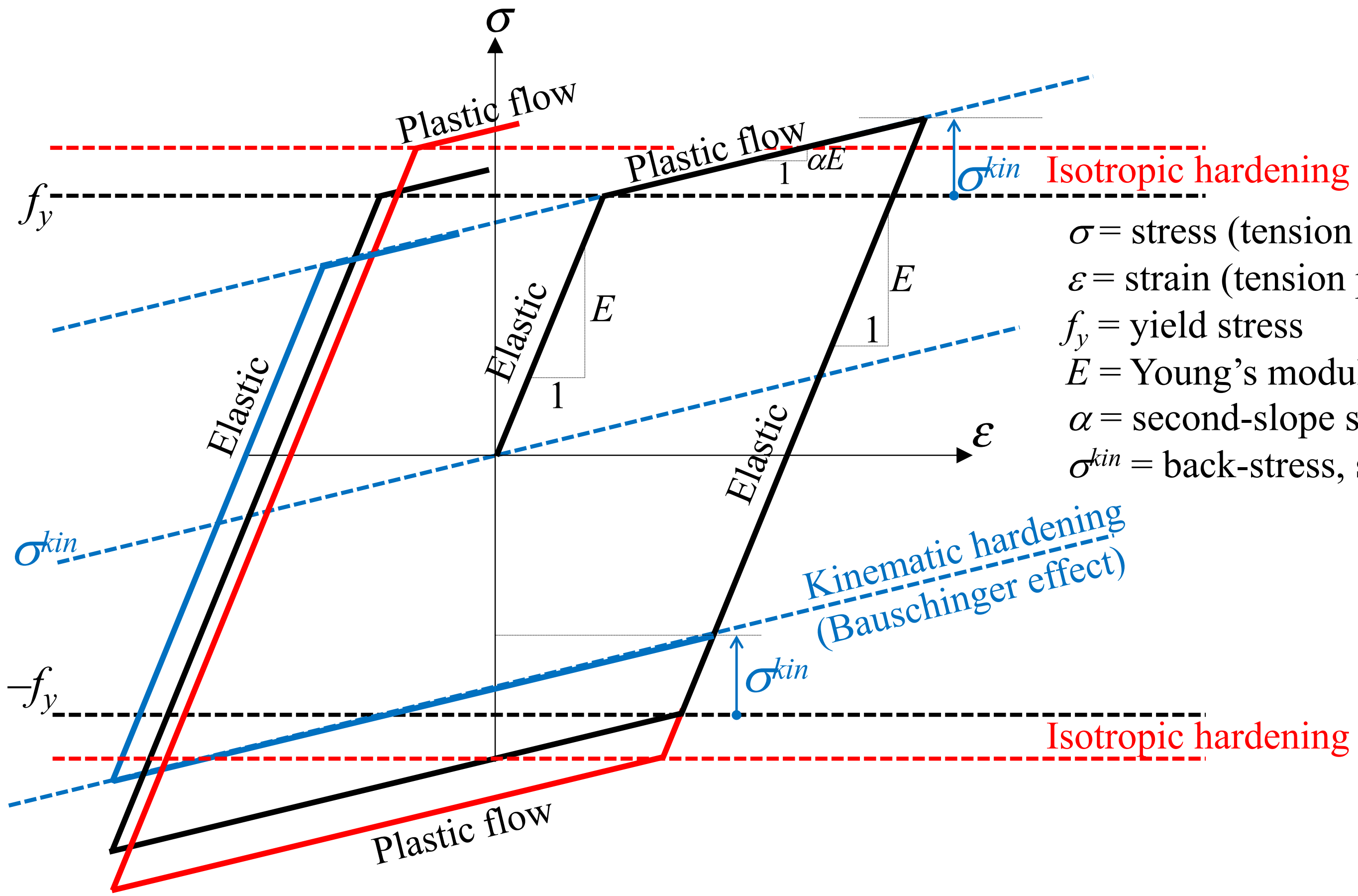
$$f = |\sigma - \sigma^{kin}| - (f_y + f^{iso})$$

- Kinematic hardening:

  Movement of elastic region

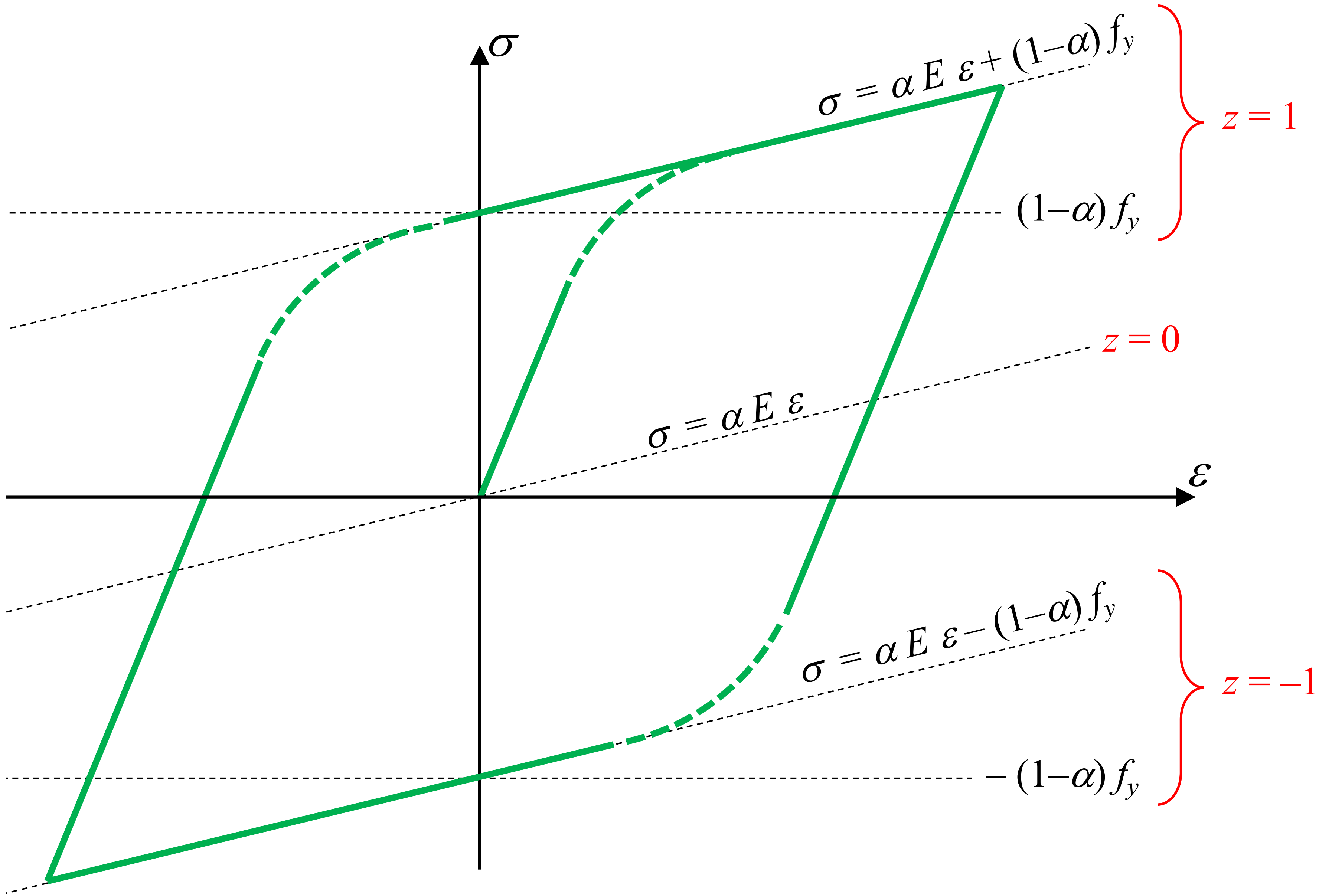
- Isotropic hardening:

  Expansion of elastic region



σ = stress (tension positive)
ε = strain (tension positive)
f_y = yield stress
E = Young's modulus
α = second-slope stiffness factor
σ^{kin} = back-stress, shift-stress

Bouc-Wen



σ
$\sigma = \alpha E \varepsilon + (1-\alpha) f_y$
$z = 1$
$(1-\alpha) f_y$
$z = 0$
$\sigma = \alpha E \varepsilon$
ε
$\sigma = \alpha E \varepsilon - (1-\alpha) f_y$
$z = -1$
$-(1-\alpha) f_y$

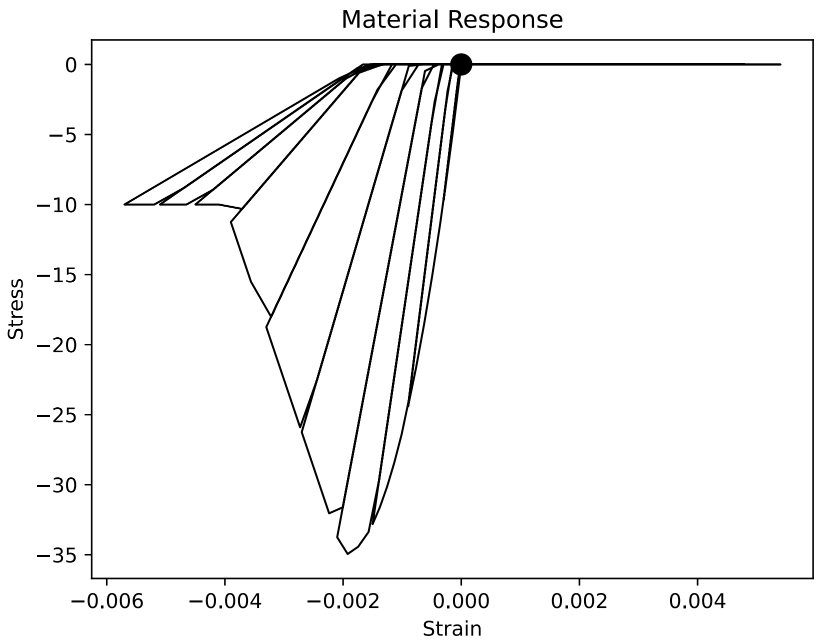
Algorithmic Tangent Modulus

- Continuum tangent: Differentiate equations

- Algorithmic tangent: Differentiate algorithm

- Newton-Raphson convergence requires the algorithmic tangent

Backbone Curve



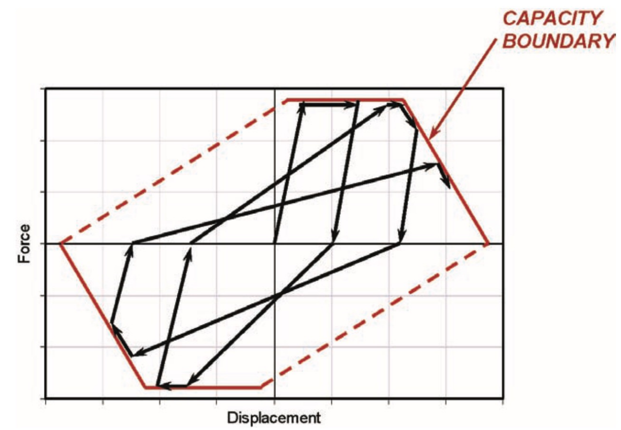


Figure 2-14 Interaction between the cyclic load path and the force-displacement capacity boundary.

2-14 2: Background Concepts FEMA P440A

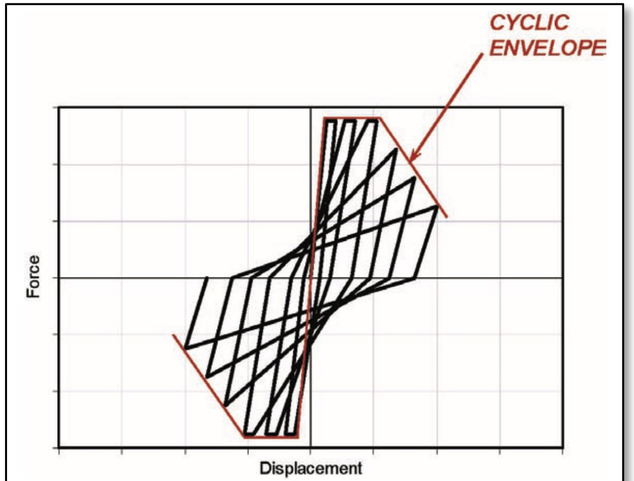


Figure 2-16 Example of a cyclic envelope.

Degradation

… in stiffness

… in strength

… between cycles

… within a cycle

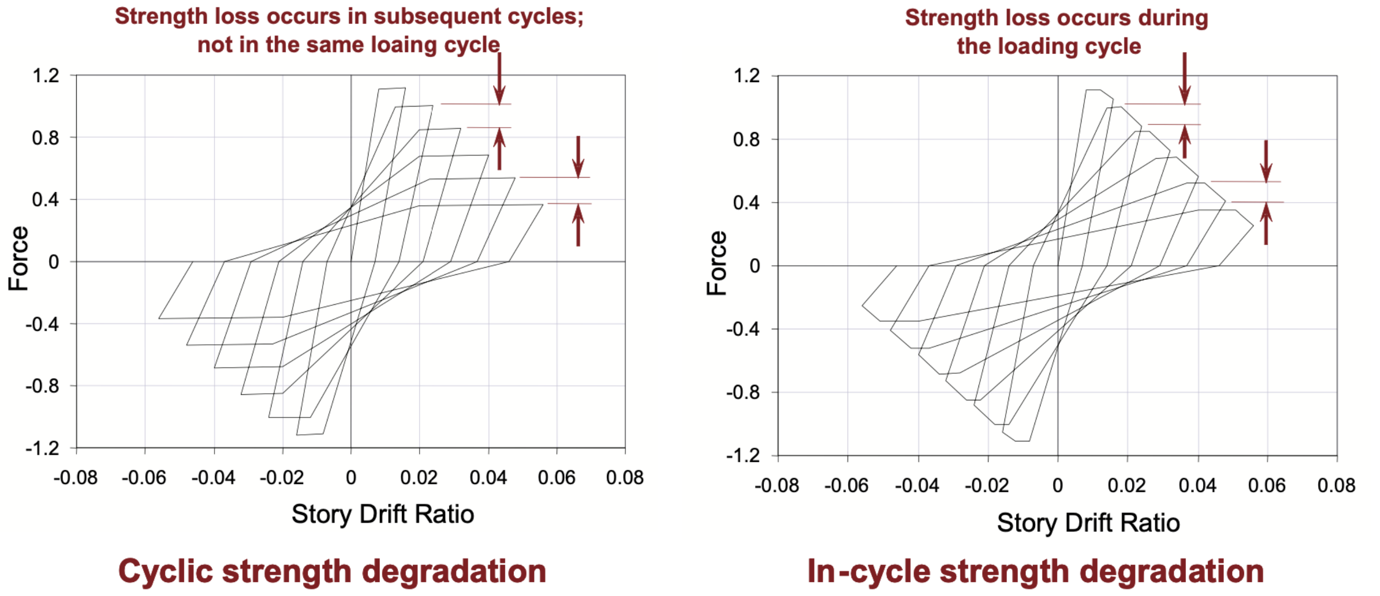


Figure 1-1 Types of degradation defined in FEMA 440.

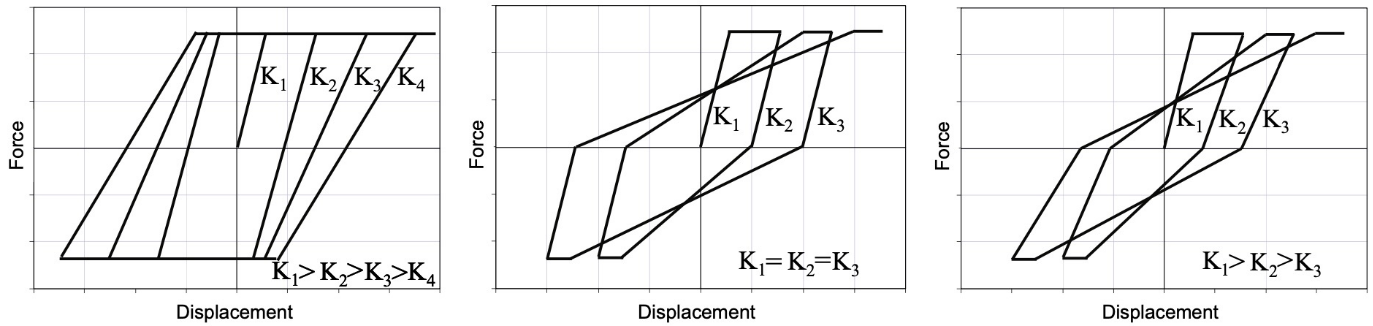


Figure 2-3 Three examples of stiffness-degrading piecewise linear hysteretic models.

Pinching

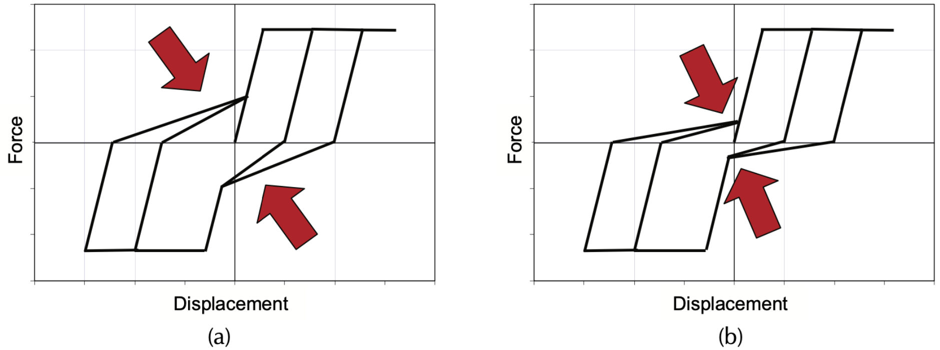
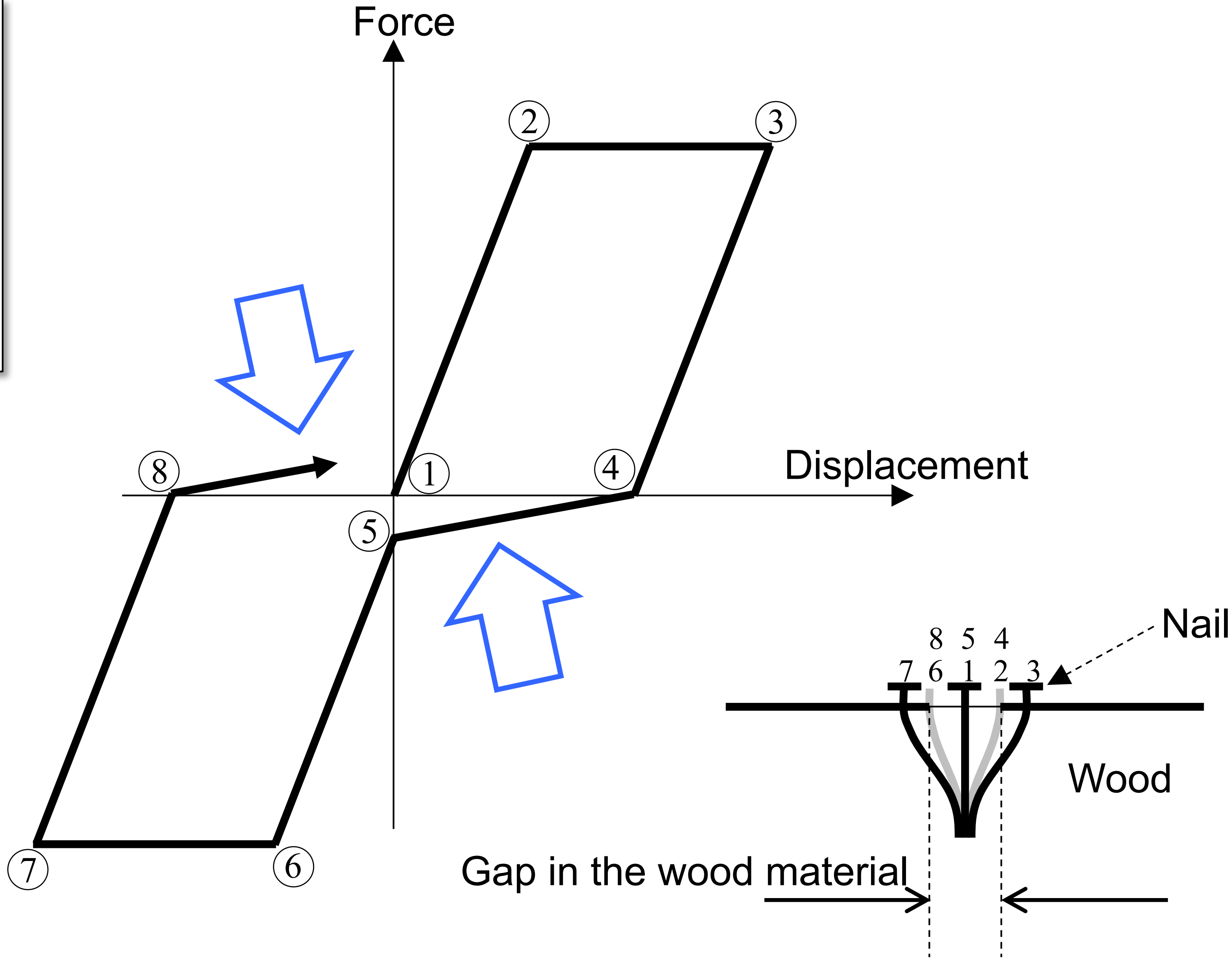


Figure 2-4 Examples of hysteretic models with: (a) moderate pinching behavior; and (b) severe pinching behavior.



More lectures:

Terje’s Toobox:

terje.civil.ubc.ca