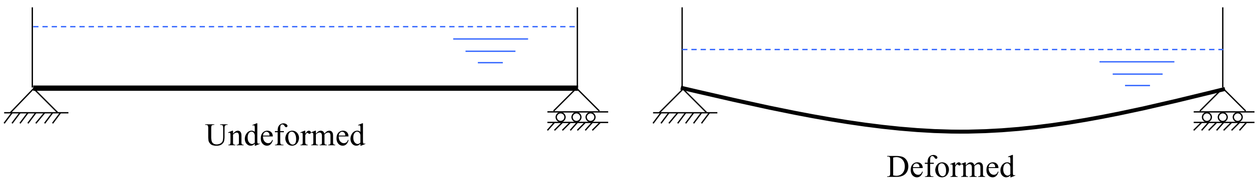
Ponding

The ponding phenomenon is illustrated in this figure, showing how the water is deeper where there is more displacement, leading to a concentration of the initially uniformly distributed load:



The problem is described by a "beam on elastic foundation" with *negative* foundation stiffness $k_s = -\rho\, g\, b$:

```
ρ = 1000;
g = 9.81;
b = 1;
ks = -ρ g b;
```

The beam is given the following properties (Units: N, m)

$$E = 9.5 \times 10^9;$$

$$I = 3\,\frac{0.038 \times 0.184^3}{12};$$

$$EI = E\, I;$$

Characteristic length:

$$lc = \sqrt[4]{-\frac{4\, EI}{ks}}$$

which yields: 3.8911

Initial water depth:

```
depth = 0.2;
```

Length of simply supported beam (notice limiting value of L=2.22 l_c, at which the displacement becomes very large, and flips to the other side for longer lengths):

```
length = 1.5 lc
```

which yields: 5.83666

Solution to basic Euler-Bernoulli differential equation:

```
solBasic =
  DSolve[{EI w''''[x] == -ρ g b depth, w[0] == 0, w[L] == 0,
    EI w''[0] == 0, EI w''[L] == 0}, w, x];
```

Solution to differential equation for beam on elastic foundation, with negative foundation stiffness:

```
solPonding =
  DSolve[{EI w''''[x] + ks w[x] == -ρ g b depth, w[0] == 0, w[L] == 0,
    EI w''[0] == 0, EI w''[L] == 0}, w, x];
```

Plot solutions:

```
Plot[{w[x] /. solBasic[[1]] /. L -> length,
  w[x] /. solPonding[[1]] /. L -> length}, {x, 0, length},
 PlotStyle → {{Black, Dashed}, {Black}},
 PlotLegends → Placed[LineLegend[{"Basic beam theory", "Ponding"}],
   {Center, Center}]]
```

which yields: