Material Nonlinearity

The material presented in this document is brief, covering mostly the grand scheme of things. For details and perhaps more clarity, check the following material posted on this website:

- Concentrated yielding
  - **Document**: Concentrated plasticity elements
  - **Python code**: G2 Element 7
  - **Python code**: G2 Element 8
  - **Python code**: G2 Element 9
  - **Python code**: G2 Element 10
- Distributed yielding
  - **Document**: Distributed plasticity elements
  - **Python code**: G2 Element 12
  - **Python code**: G2 Element 13
- Material models
  - **Document**: Uniaxial Plasticity Material Model
  - **Document**: Bouc-Wen Material Model
  - **Python code**: G2 Bilinear Material Model
  - **Python code**: G2 Plasticity Material Model
  - **Python code**: G2 BoucWen Material Model

In contrast to geometric nonlinearity, material nonlinearity is intuitively understood by the beginner structural engineer: When a material is stressed hard enough, it yields or cracks in an irreparable manner, and small additional forces lead to large displacements. A nonlinear force-displacement "pushover" curve for a building is intuitively understood to be nonlinear because the materials in the columns and beams are damaged. The stress-strain and force-displacement behaviours depend on the material type. However, some modelling concepts are generic, and addressed in this document.

Yielding

Yielding is the most fundamental of characteristics of nonlinear material behaviour. A bi-linear stress-strain curve that exhibits a kink at the yield stress is the basic visualization. The document on stress-based failure criteria suggests Tresca, von Mises, and Drucker-Prager as prominent criteria for when the material enter into a yielding state. J2 plasticity theory for uniaxial and multi-axial stress states provides additional modelling details for the evolution of yielding as the material experiences further deformation.

Hysteresis

Hysteretic behaviour means that the state of the material depends on the stress-strain history it has undergone. To capture hysteretic behaviour, it is necessary to implement "incremental" material models that stores "history variables" whenever a converged equilibrium state is reached. At every state determination, an incremental material model receives the strain increment, i.e., the increment in trial strain from the previously committed equilibrium state. Next, the state determination in the material proceeds by determining the new stress associated with that strain increment, utilizing one or more history variables that were stored at the previously committed state. An example of a history variable is the back-stress, i.e., shift-stress that captures kinematic hardening of a bi-linear uniaxial material model. When the material model determines its new state, i.e., the new stress associated with the given strain increment, it employs “loading and unloading rules.” For example, a negative strain increment may lead the material to apply an unloading rule that implies elastic unloading with stiffness equal to the initial elastic stiffness. Such an unloading rule is common in the modelling of uniaxial steel behaviour. In other words, an incremental material model addresses hysteretic material behaviour by means of an array of if-statements in its computer implementation.

Hardening

After a material yields it is sometimes assumed that it exhibits zero additional stress as the deformation increases. That is known as elasto-plastic analysis, addressed elsewhere on this website with the lower- and upper-bound theories of elasto-plastic capacity analysis, often labelled simply plastic capacity analysis. In reality, the inclusion of hardening, i.e., the addition of stress as the strain increases, is often warranted. As described in the document on uniaxial plasticity, the hardening can be modelled as isotropic (expanding elastic stress region) and/or kinematic (moving elastic stress region).

Backbone Curve

When a material is subjected to multiple strain cycles, i.e., multiple loading and unloading events, the result can be presented as a hysteresis curve. A simple case is presented in Figure 1. The envelope of the hysteresis curve is called a “backbone curve.”

Degradation

The hysteresis curve that results from multiple loading and unloading events may exhibit diminishing stiffness and/or strength from once cycle to the next. This is called degradation. Combined stiffness and strength degradation is not uncommon. Particular to the strength, it may diminish from one cycle to the next, or during a cycle. Those options are labelled cyclic and in-cycle degradation, respectively.

Pinching

When cracks are opening and closing during load cycles, either in wood, concrete, or masonry, it is common to observe “pinching behaviour.” The word pinching comes from the pinched force-displacement curve illustrated by red arrows in Figure 1. That figure illustrates pinching behaviour in the context of a nail that is being forced back-and-forth in a wood material. Points 4 and 8 are important for understanding the pinching behaviour; at those points the nail separates from the wood material and experiences essentially zero stiffness, except for the stiffness of the nail itself, until it meets wood material again on the other side of the gap in the wood material. That gap opens up because the nail is pressing against the wood, essentially causing irreversible yielding of the material.

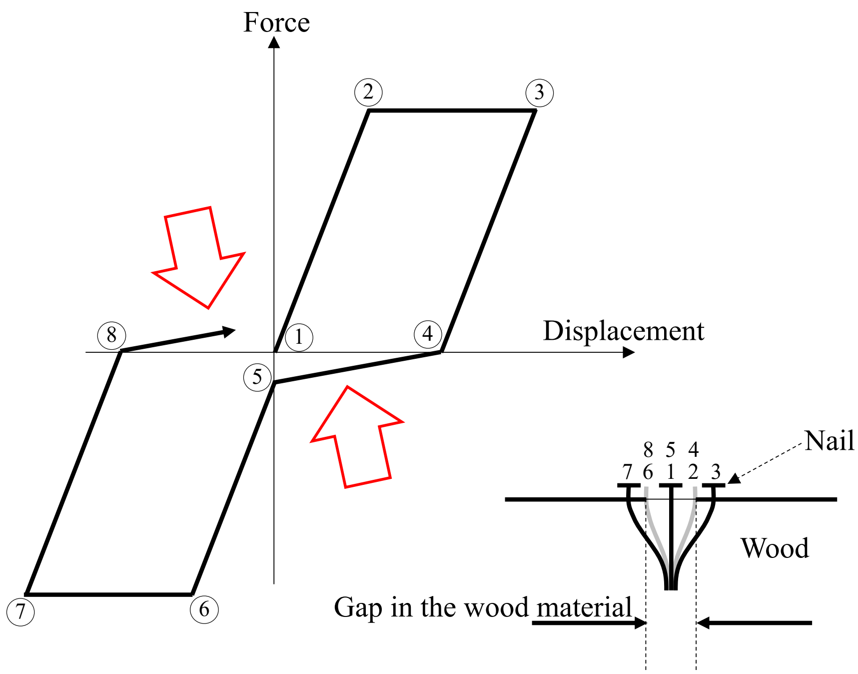

Figure 1: Pinching behaviour.