

Invariance Problem

In order to visualize the invariance problem of the mean-centred first-order second-moment reliability method, considering the following limit-state functions:

$$g1 = R - S;$$

$$g2 = \text{Log}\left[\frac{R}{S}\right];$$

$$g3 = 1 - \frac{S}{R};$$

Gradient vectors:

$$\nabla g1 = \{D[g1, R], D[g1, S]\};$$

$$\nabla g2 = \{D[g2, R], D[g2, S]\};$$

$$\nabla g3 = \{D[g3, R], D[g3, S]\};$$

Covariance matrix:

$$\Sigma = \{\{sR^2, \rho_{RS} sR sS\}, \{\rho_{RS} sR sS, sS^2\}\};$$

Second-moment information:

$$\mu_R = 30;$$

$$\mu_S = 20;$$

$$\sigma_R = 5;$$

$$\sigma_S = 10;$$

$$\rho = 0;$$

$$\text{info} = \{R \rightarrow \mu_R, S \rightarrow \mu_S, sR \rightarrow \sigma_R, sS \rightarrow \sigma_S, \rho_{RS} \rightarrow \rho\};$$

MCFOSM reliability indices:

$$\beta_1 = \left(\frac{g1}{\sqrt{\nabla g1 \cdot \Sigma \cdot \nabla g1}} \right) /. \text{info} // N$$

which yields: 0.894427

$$\beta_2 = \left(\frac{g_2}{\sqrt{\nabla g_2 \cdot \Sigma \cdot \nabla g_2}} \right) /. \text{info} // N$$

which yields: 0.769316

$$\beta_3 = \left(\frac{g_3}{\sqrt{\nabla g_3 \cdot \Sigma \cdot \nabla g_3}} \right) /. \text{info} // N$$

which yields: 0.948683

Linearized limit-state functions:

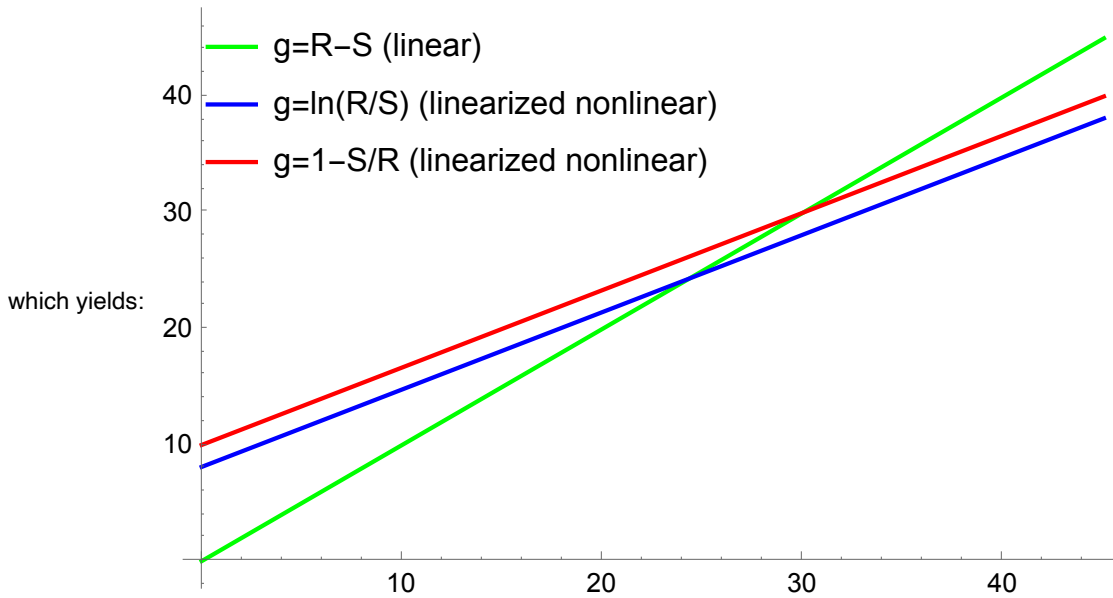
```
g1Lin = (g1 + ∇g1.({r, s} - {R, S})) /. info /. {r -> R, s -> S};
g2Lin = g2 + ∇g2.({r, s} - {R, S}) /. info /. {r -> R, s -> S};
g3Lin = g3 + ∇g3.({r, s} - {R, S}) /. info /. {r -> R, s -> S};
```

Solve linearized limit-state functions for S:

```
s1 = Solve[g1Lin == 0, S];
s2 = Solve[g2Lin == 0, S];
s3 = Solve[g3Lin == 0, S];
```

Plot limit-state surfaces in the original variable space:

```
Plot[{S /. s1, S /. s2, S /. s3}, {R, 0, 1.5 μR},
PlotStyle → {Green, Blue, Red},
PlotLegends →
Placed[
LineLegend[{"g=R-S (linear)", "g=ln(R/S) (linearized nonlinear)",
" g=1-S/R (linearized nonlinear)"}], {Left, Top}]]
```



Prepare for transformation into standard space, by first establishing the mean vector:

$$M = \{\mu_R, \mu_S\};$$

Then the Cholesky decomposition of the covariance matrix:

$$L = \text{CholeskyDecomposition}[\Sigma /. \text{info}]^T;$$

Second-moment transformation:

$$X = M + L \cdot \{Y1, Y2\};$$

Transform limit-state functions into the standardized space:

$$G1 = g1 / . \{R \rightarrow X[[1]], S \rightarrow X[[2]]\};$$

$$G2 = g2 / . \{R \rightarrow X[[1]], S \rightarrow X[[2]]\}$$

$$G3 = g3 / . \{R \rightarrow X[[1]], S \rightarrow X[[2]]\};$$

which yields: $\text{Log}\left[\frac{30 + 5 Y1}{20 + 10 Y2}\right]$

Gradient vectors:

$$\nabla G1 = \{D[G1, Y1], D[G1, Y2]\};$$

$$\nabla G2 = \{D[G2, Y1], D[G2, Y2]\}$$

$$\nabla G3 = \{D[G3, Y1], D[G3, Y2]\};$$

which yields: $\left\{\frac{5}{30 + 5 Y1}, -\frac{10}{20 + 10 Y2}\right\}$

Linearize the limit-state functions in the standardized space:

$$G1Lin = (G1 + \nabla G1 \cdot (\{y1, y2\} - \{Y1, Y2\})) / . \{Y1 \rightarrow 0, Y2 \rightarrow 0\};$$

$$G2Lin = (G2 + \nabla G2 \cdot (\{y1, y2\} - \{Y1, Y2\})) / . \{Y1 \rightarrow 0, Y2 \rightarrow 0\};$$

$$G3Lin = (G3 + \nabla G3 \cdot (\{y1, y2\} - \{Y1, Y2\})) / . \{Y1 \rightarrow 0, Y2 \rightarrow 0\};$$

Solve standardized limit-state functions for Y2:

$$s1s = \text{Solve}[G1Lin == 0, y2];$$

$$s2s = \text{Solve}[G2Lin == 0, y2]$$

$$s3s = \text{Solve}[G3Lin == 0, y2];$$

which yields: $\left\{\left\{y2 \rightarrow \frac{1}{3} \left(y1 + 6 \text{Log}\left[\frac{3}{2}\right]\right)\right\}\right\}$

Plot limit-state surfaces in the standardized variable space:

```

plotBound = 1.5;
Plot[{y2 /. s1s, y2 /. s2s, y2 /. s3s}, {y1, -plotBound, plotBound},
  PlotRange -> {{-plotBound, plotBound}, {-plotBound, plotBound}},
  PlotStyle -> {Green, Blue, Red},
  PlotLegends -> Placed[LineLegend[{"g=R-S", "g=ln(R/S)", "g=1-S/R"}],
    {Left, Bottom}]]

```

