# Confidence in Number of Ground Motions

Suppose you are conducting nonlinear dynamic analyses of a building archetype. The recorded ground motions that you apply as loading are scaled to two hazard levels: 1.0 times the uniform hazard spectrum (UHS) and 2.0 times UHS.

The question is how many structural collapses, or equivalently, numerical instability, to permit in suites of 11, 20, 50, and 100 ground motions. For the sake of this problem, your target failure probability is 10% at 1·UHS and 50% at 2·UHS.

Those targets nominally imply that you would consider 10 collapses acceptable in 100 analyses at 1·UHS, 50 collapses in 100 analyses at 1·UHS, and so forth. However, while keeping those target probabilities, you wish to tighten that requirement in order to increase your "confidence" in the actual failure probabilities below the targets. Specifically, how many collapses would you accept, for the various suite sizes, at each hazard level, in order to have a 90% chance, or a 95% chance, that the failure probability is below the targets?

This problem can be solved by adopting Bernoulli trials as the governing stochastic model. That model governs independent trials, with outcomes *success* or *failure*. In this case, for mathematical convenience, let success denote collapse caused by a ground motion. The Bernoulli model parameter, $p$, denotes the probability of success in each trial.

Bayesian statistical inference is available for updating the probability distribution for $p$, given observations of the number of successes, $x$, in $n$ trials. In other words, by doing Bayesian updating, we get a probability distribution for $p$. In turn, that allows us to suggest how many failures can be accepted in order to have a 90% or 95% chance that $p$ is below 10% and 50%. Trying different number of observed failures leads to the following table:

| | 11 analyses | 20 analyses | 50 analyses | 100 analyses |
|---|---|---|---|---|
| Acceptable number of failures to keep P($p$<0.5) above 90% | 3 | 7 | 20 | 43 |
| Acceptable number of failures to keep P($p$<0.5) above 95% | 2 | 6 | 19 | 41 |
| Acceptable number of failures to keep P($p$<0.1) above 90% | 0 | 0 | 2 | 6 |
| Acceptable number of failures to keep P($p$<0.1) above 95% | 0 | 0 | 1 | 5 |