Plastic Capacity Analysis by Hand

The purpose of plastic capacity analysis is to determine the ultimate capacity of cross-sections (lower-bound theorem) and of a global frame/plate structure (upper-bound theorem). The label elasto-plastic is sometimes given to this type of analysis because the stress-strain relationship is assumed to be as shown in Figure 1. That means elastic up to yielding, and no increase in stress thereafter, regardless of the amount of strain. Plastic capacity analysis is suitable for quick hand calculations to obtain the capacity; however, it is important to note that the deformations remain unknown. Excessive displacements and rotations may develop before the ultimate capacity is reached. In other words, elasto-plastic analysis, i.e., plastic capacity analysis is suitable in applications where only the ultimate capacity is sought and the amount of damage is irrelevant.

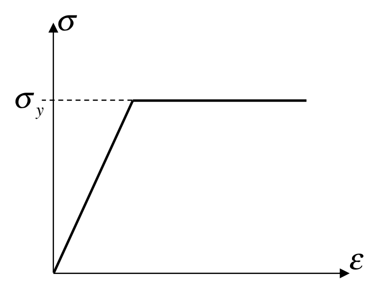

Figure 1: Elasto-plastic material model.

Lower-bound Theorem

When plastic capacity analysis is applied to cross-sections, in order to obtain the ultimate capacity of the cross-section to carry moment and/or axial force, we essentially try to guess how the stresses are distributed over the cross-section. Full yield stress is assumed everywhere, as exemplified by the gray-shaded stress blocks in Figure 2. Some stress blocks are reserved for carrying the moment, another to carry the axial force. The lower-bound theorem of plastic capacity analysis states:

> *A selected stress distribution will, when equilibrium is established between the stress blocks and the stress resultants, produce a capacity that is less than or equal to the correct value.*

That means the moment/axial force capacities that are obtained are either correct or too low. That is helpful in design, because the capacity estimates are correct or conservative.

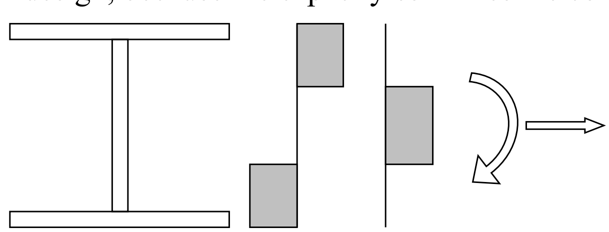

Figure 2: Elasto-plastic material model.

Upper-bound Theorem

The lower-bound theorem can also be applied to determine the ultimate capacity of a structure, such as a frame, to carry applied loads. That is addressed in a document on computational plastic capacity analysis. However, for hand calculations the upper-bound theorem is more suitable for determining the ultimate capacity of frames and slabs. In this approach, we try to guess the deformed shape, called "mechanism" of the structure, when it has reached its ultimate capacity. That implies guessing the location of plastic hinges for frames and plastic yield lines for slabs. The upper-bound theorem then states:

> *A selected kinematic mechanism will, when internal work is set equal to external work, produce a capacity that is greater than or equal to the correct value.*

That means the calculated capacity is either correct or too high. That is dangerous, because it means the estimate is either correct or unconservative. Hence, in plastic capacity analysis with the upper-bound theorem we must determine the capacity for as many mechanisms as we can muster. The actual capacity is the smallest them, or smaller. A document with examples is posted on this website to better understand the application of plastic capacity analysis.