

Cable Cases

Parabola

In the document on cables and catenary shapes, posted near this file, a solution is obtained for the case of a parabola for which the three heights H_1 , H_2 , and H_3 are given. They are here specified in metres, together with a length, and a distributed load given in kN/m:

$$H_1 = 15;$$

$$H_2 = 8;$$

$$H_3 = 10;$$

$$L = 30;$$

$$q = 3;$$

The constants that appear in the solution to this problem are:

$$c_1 = \frac{(H_1 - 2H_2 + H_3) L^2 q - 2 \sqrt{-(H_1 - H_2)(H_2 - H_3) L^4 q^2}}{2 (H_1 - H_3)^2};$$

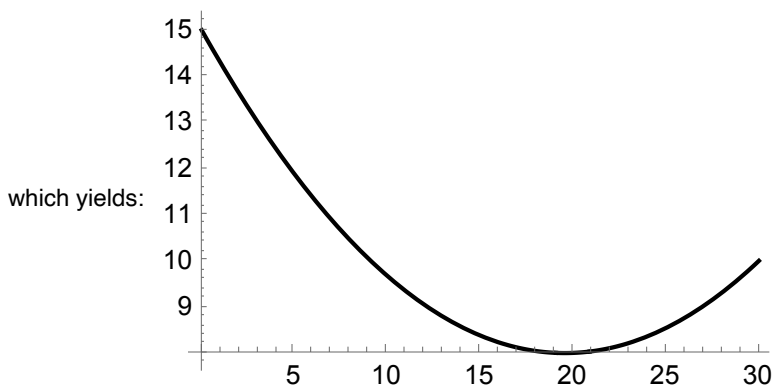
$$c_2 = \frac{(-H_1 + H_2) L^2 q + \sqrt{-(H_1 - H_2)(H_2 - H_3) L^4 q^2}}{(H_1 - H_3) L};$$

$$c_3 = H_1;$$

The solution is here plotted:

$$y = \frac{1}{2} \frac{q x^2}{c_1} + \frac{c_2}{c_1} x + c_3;$$

`Plot[y, {x, 0, L}, PlotStyle -> Black]`



The axial force in the cable, together with its horizontal and vertical components, are plotted here:

$$dydx = D[y, x];$$

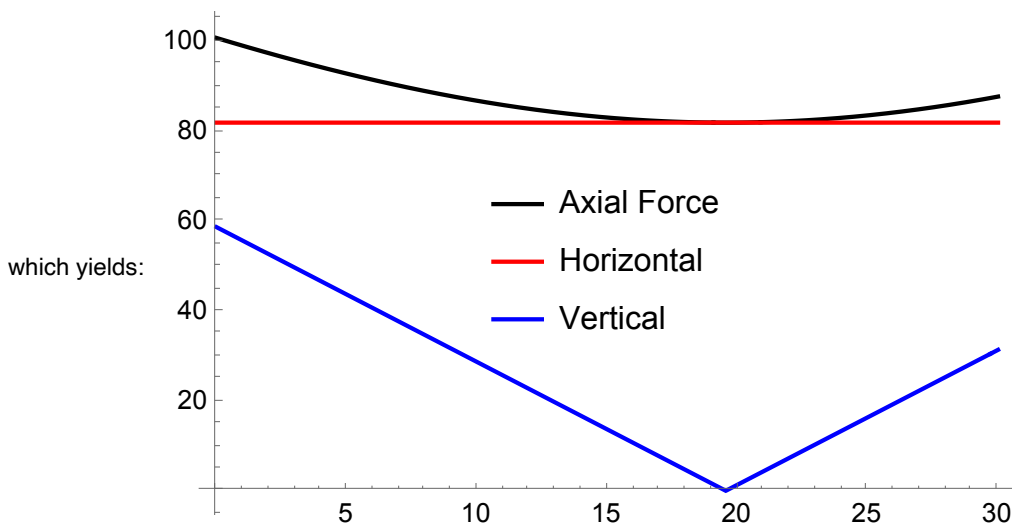
$$\theta = \text{ArcTan}[dydx];$$

$$T = \frac{c1}{\text{Cos}[\theta]};$$

$$F_{\text{horz}} = c1;$$

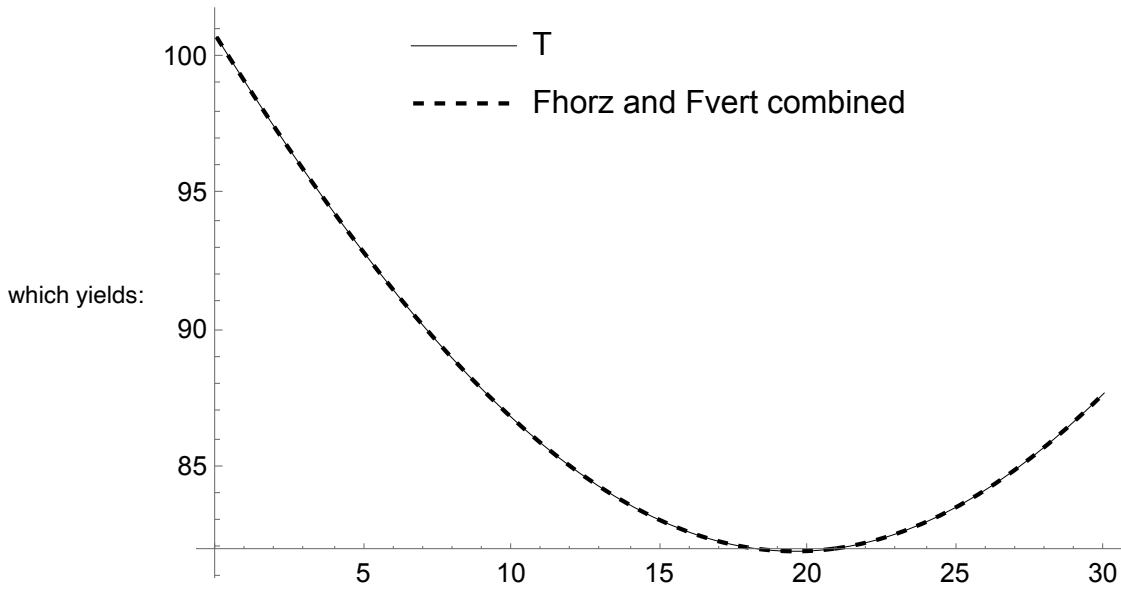
$$F_{\text{vert}} = \text{Abs}[q x + c2];$$

```
Plot[{T, Fhorz, Fvert}, {x, 0, L}, PlotStyle -> {Black, Red, Blue},
PlotLegends ->
  Placed[LineLegend[{"Axial Force", "Horizontal", "Vertical"}],
  {Center, Center}]]
```



The next plot verifies that the horizontal and vertical force components plotted above gives the total axial force along the cable when combined:

```
Plot[{T,  $\sqrt{F_{horz}^2 + F_{vert}^2}$ }, {x, 0, L},
PlotStyle -> {{Black, Thin}, {Black, Dashed}},
PlotLegends -> Placed[LineLegend[{"T", "Fhorz and Fvert combined"}],
{Center, Top}]]
```



Catenary

For the catenary case, it is harder to input locations of points on the cable; that requires the solution of a nonlinear equation. Instead, the following results are shown simply to visualize one arbitrary case. Notice that the end points of the cable are selected to be symmetric about the origin. Also notice that the constant c gives the height of the cable at the origin:

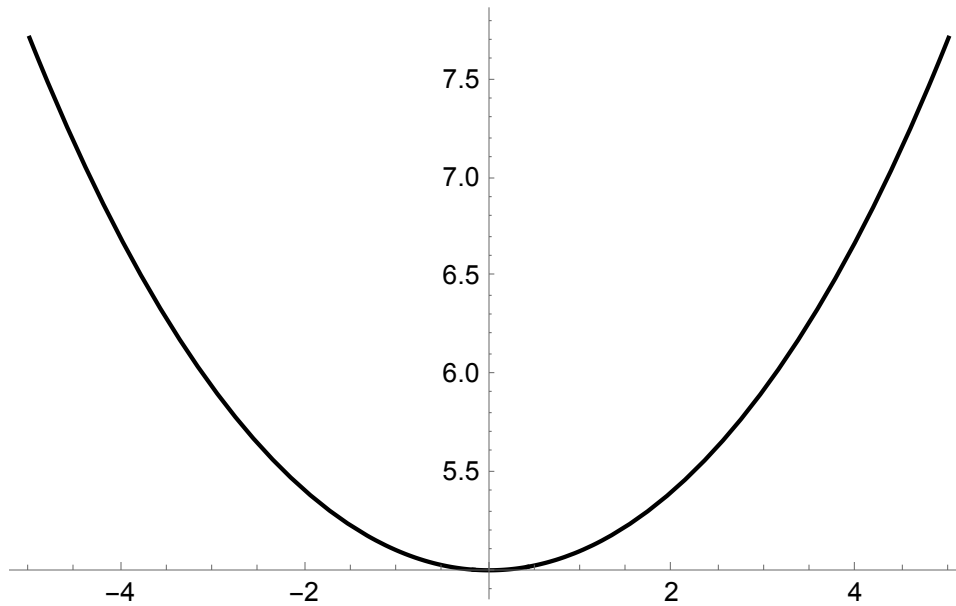
$$x_{End} = 5;$$

$$c = 5;$$

$$y = c \operatorname{Cosh}\left[\frac{x}{c}\right];$$

```
Plot[y, {x, -xEnd, xEnd}, PlotStyle -> Black]
```

which yields:



From the document on cables and catenary shapes, posted near this file, the forces in the cable are, for mass density per unit length given in kg/m for a circular cross-section shape with radius r :

$$r = 0.015;$$

$$\rho = 7850 \pi r^2;$$

$$g = 9.81;$$

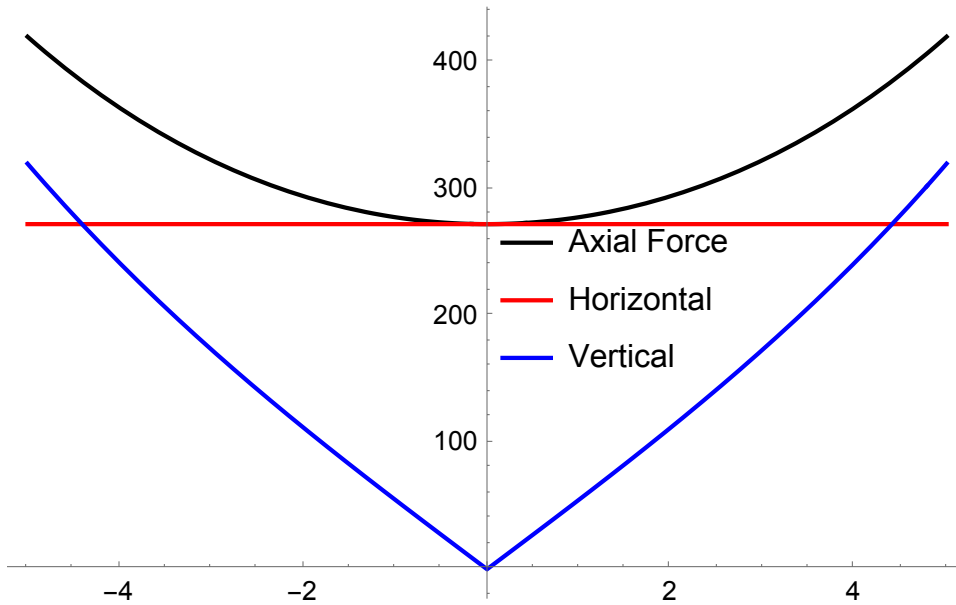
$$T = \rho g y;$$

$$F_{\text{horz}} = \rho g c;$$

$$F_{\text{vert}} = \text{Abs} \left[\rho g c \text{Sinh} \left[\frac{x}{c} \right] \right];$$

```
Plot[{T, Fhorz, Fvert}, {x, -xEnd, xEnd}, PlotStyle -> {Black, Red, Blue},
PlotLegends ->
  Placed[LineLegend[{"Axial Force", "Horizontal", "Vertical"}],
  {Center, Right}]]
```

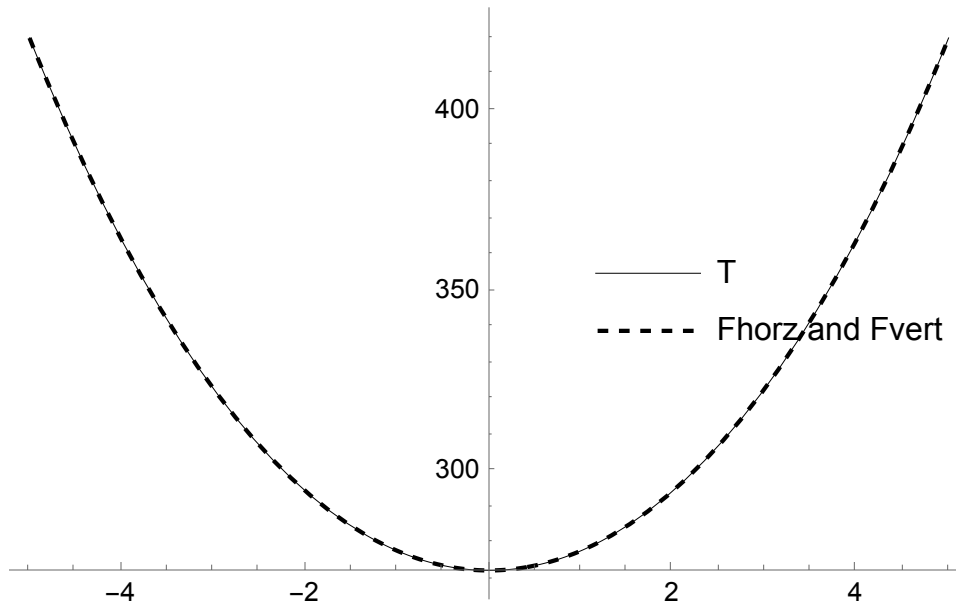
which yields:



Again the combination of the horizontal and vertical components can be verified:

```
Plot[{T,  $\sqrt{F_{horz}^2 + F_{vert}^2}$ }, {x, -xEnd, xEnd},
PlotStyle -> {{Black, Thin}, {Black, Dashed}},
PlotLegends -> Placed[LineLegend[{"T", "Fhorz and Fvert"}],
{Right, Center}]]
```

which yields:



Comparison of Parabola and Catenary

In order to match the catenary, consider the following input to the parabola:

```
H1 = y /. x -> -xEnd;
H2 = c;
H3 = y /. x -> xEnd;
L = 2 xEnd;
q =  $\rho g$ ;
```

Recalculate parabola and plot:

$$c1 = \frac{L^2 q}{4 (H1 - 2 H2 + H3)} ;$$

$$c2 = \frac{-3 H1 L q + 4 H2 L q - H3 L q}{4 (H1 - 2 H2 + H3)} ;$$

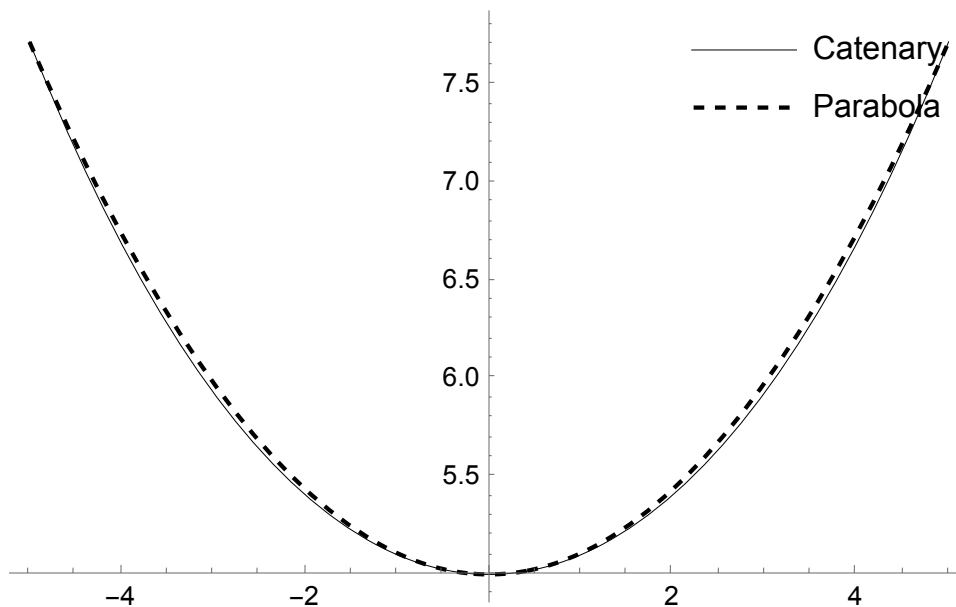
$$c3 = H1 ;$$

$$yParabola = \frac{1}{2} \frac{q}{c1} x^2 + \frac{c2}{c1} x + c3 ;$$

$$yParabolaShifted = yParabola /. x -> \left(x + \frac{L}{2} \right) ;$$

```
Plot[{y, yParabolaShifted}, {x, -xEnd, xEnd},
  PlotStyle -> {{Black, Thin}, {Black, Dashed}},
  PlotLegends -> Placed[LineLegend[{"Catenary", "Parabola"}],
    {Right, Top}]]
```

which yields:



Also compare axial forces:

```
dydx = D[yParabola, x];
```

```
θ = ArcTan[dydx];
```

```
Tparabola =  $\frac{c1}{\text{Cos}[\theta]}$ ;
```

```
FhorzParabola = c1;
```

```
FvertParabola = Abs[q x + c2];
```

```
TparabolaShifted = Tparabola /. x ->  $\left(x + \frac{L}{2}\right)$ ;
```

```
FhorzParabolaShifted = FhorzParabola /. x ->  $\left(x + \frac{L}{2}\right)$ ;
```

```
FvertParabolaShifted = FvertParabola /. x ->  $\left(x + \frac{L}{2}\right)$ ;
```

```
Plot[{T, Fhorz, Fvert, TparabolaShifted, FhorzParabolaShifted,
      FvertParabolaShifted}, {x, -xEnd, xEnd},
PlotStyle -> {{Black, Thin}, {Red, Thin}, {Blue, Thin}, Black, Red, Blue},
PlotLegends ->
  Placed[LineLegend[{"T cat.", "Fhorz cat.", "Fvert cat.",
                    "T par.", "Fhorz par.", "Fvert par."}], {Top, Center}]]
```

