Stress & Strain

The concept of stress is important in modern structural engineering, because stress serves as a design criterion. For example, in a steel component, we do not allow the stress to approach the yield stress of the material. For some materials, such as reinforced concrete, local stress values are hard to calculate. In such cases, the concept of "stress resultant" is employed. That means bending moment, shear force, and axial force. Most design codes display capacity formulas formulated in terms of such stress resultants. So, what is stress? In essence, it is force per unit area. For example, a 10kN force, i.e., roughly a metric ton, uniformly distributed over a 10cm^2 area implies a 10N/mm^2 stress, i.e., 10MPa. One MPa is 145.038 pounds per square inch, psi. A typical yield stress for steel is 350MPa. A typical crushing strength for concrete is 35MPa.

Continuum Assumption

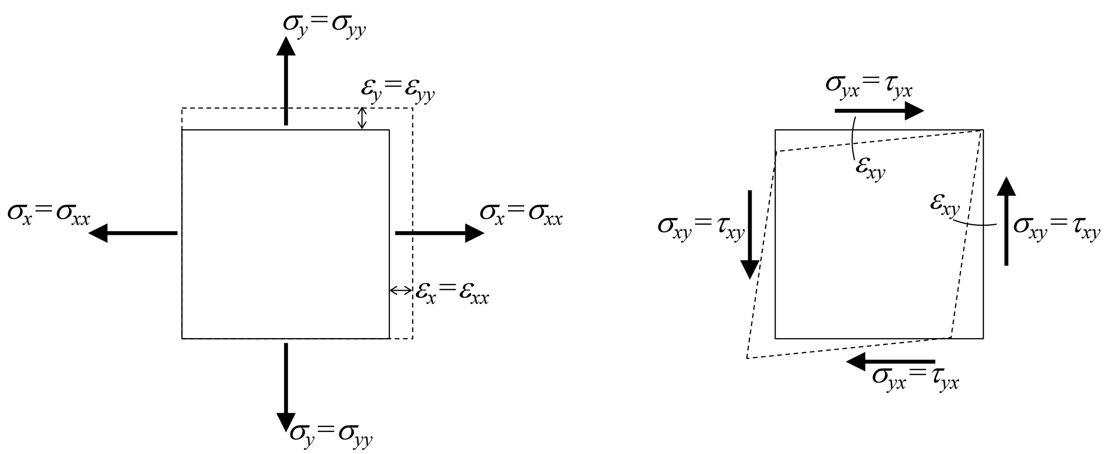
Notice that the smearing of force over an area assumes the material to be an uninterrupted continuum. In work by Navier and others, preceding Cauchy's introduction of the concept of stress, the internal forces in materials were modelled in terms of molecular forces. Conversely, the use of stress and strain implies a continuum assumption. In other words, the material properties are assumed to be "smeared" continuously through the materials. This is never completely true, but it is almost always a helpful assumption. For materials like steel, aluminum, and titanium the continuum assumption is particularly accurate and the calculation of stress and stress appear natural. The introduction of such materials is perhaps one reason why the concept of stress become so popular. In fact, "allowable stress" was the dominant design paradigm for many decades after it was introduced nearly a century ago. Conversely, the continuum assumption comes with caveats for reinforced concrete. Difficulties associated with the calculation of stress in reinforced concrete, where approximate stress blocks are often assumed, is one reason why the allowable stress paradigm has been amended by limit-state design, where limit-states beyond stress are checked, such as the stress resultants mentioned above.

Stress and Beam Theory

It is interesting to note that the beam theory we today name after Leonard Euler (1707-1783) and Daniel Bernoulli (1700-1782), and to some extent Daniel's uncle Jacob Bernoulli (1654-1705), came about before the concepts of stress and strain existed. While Euler and Bernoulli established the beam theory with the correct neutral axis it was Augustin Cauchy (1789-1857) who first introduced the concepts of stress and strain in 1822. Although Antoine Parent (1666-1716) and later Charles-Augustin de Coulomb (1736-1806) used similar concepts in beam theory, earlier work essentially considered the molecular forces between individual particles rather than stress distributed in a solid continuum. Cauchy took the continuum idea from hydrodynamics and the concepts stress and strain in solids are now omnipresent in solid mechanics and structural engineering.

2D Coordinate Stress and Strain

Stresses and strains are often visualized for infinitesimally small material particles with square shape and dimensions dx, dy, and dz, oriented in the x, y, z coordinate system. Stresses that act in those directions are called coordinate stresses. Figure 1 shows coordinate stresses, in the form of axial stress on the left-hand side and shear stress on the right-hand side. Oftentimes, axial and shear stress appear simultaneously, but in this figure, they are separated. The stresses in Figure 1 are coordinate stresses, because they act in the x and y directions. σ is the common symbol for stress, sometimes changed to τ for shear stress. ε is the common symbol for strain, with γ is used for the "engineering shear strain," which equals $2\varepsilon_{xy}$ as shown on the right-hand side of Figure 1. In other documents posted on this website, care is taken to let the first index of stress indicate the direction of the vector that is normal to the surface on which the stress acts; the second index is the direction of the stress itself.



Axial stress and strain

Shear stress and strain ($\gamma_{yx} = 2\varepsilon_{xy}$)

Figure 1: Coordinate stress and strain for 2D material particle.

3D Coordinate Stress and Strain

Figure 2 shows the stress components for an infinitesimally small cube of a continuum material. Again, the first index indicates the direction of the normal vector to the plane where the stress acts and the second index indicates the direction of the stress. Axial stresses are positive in tension and shear stresses are positive when they act in the positive axis direction on a surface that has a positive axis direction as the surface normal. In Figure 2, the symbol σ is used for axial stress and τ for shear stress. When σ is used for both, then equal indices are understood to mean axial stresses and different indices mean shear stress:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \equiv \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \tag{1}$$

written σ_{ij} in index notation. Similarly, the matrix of strains is

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & 0.5\cdot\gamma_{xy} & 0.5\cdot\gamma_{xz} \\ 0.5\cdot\gamma_{yx} & \varepsilon_{yy} & 0.5\cdot\gamma_{yz} \\ 0.5\cdot\gamma_{zx} & 0.5\cdot\gamma_{zy} & \varepsilon_{zz} \end{bmatrix} \tag{2}$$

written ε_{ij} in index notation.

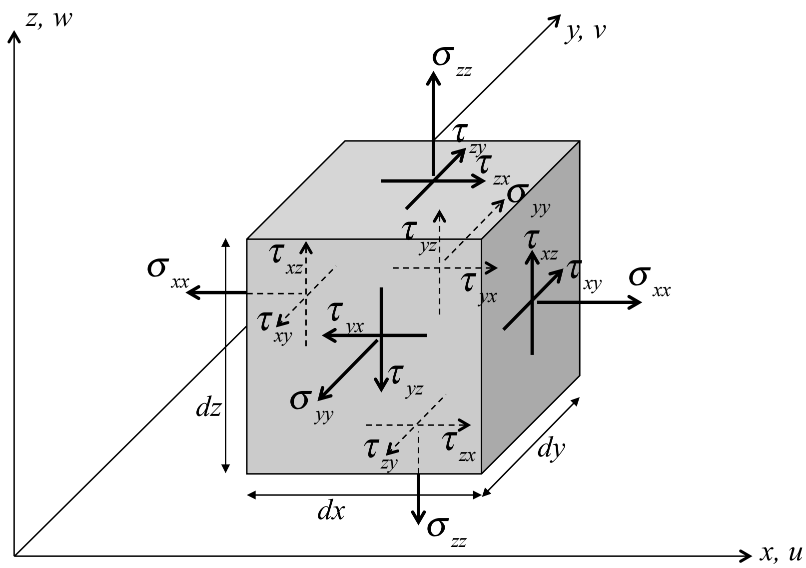


Figure 2: Components of coordinate stress.

The matrices in Eqs. (1) and (2) are referred to as the stress tensor and the strain tensor, respectively. Formulations with more extensive used of tensors are found in the continuum mechanics document posted near this one. An alternative to the tensor notation is called Voight notation, in which the stress and strain components are collected in vectors:

$$\boldsymbol{\sigma} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} \qquad \boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \tag{3}$$