Boundary Value Problems

The objective in structural and general mechanical problems is to determine displacements, and also stresses and strains in the material, due to external loads. In order to achieve that objective, it is necessary to consider equilibrium and kinematic compatibility, as well as a material law that governs the relationship between stresses and strains. These ingredients are displayed in Figure 1, showing that equilibrium equations relate the externally applied loads to stresses in the material, and showing that kinematic compatibility equations relate the global displacements to the strains in the material. This is the reason why the documents posted near this one are organized into Kinematic Compatibility; Material Law; and Equilibrium. In mathematical jargon, the ingredients in Figure 1 form a boundary value problem. A fourth ingredient is required in order to find solutions to specific problems; namely, problem-specific boundary conditions. The boundary conditions come in the form of restrains on the displacements, called essential boundary conditions as explained elsewhere, and in the form of applied loads, called natural boundary conditions.

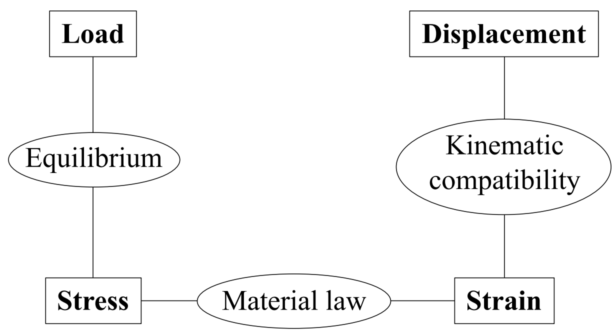

Figure 1: Ingredients of boundary value problems in mechanics.

Figures similar to Figure 1 are presented in other documents on this website, addressing various specific boundary conditions. When addressing "structural" boundary value problems, i.e., truss and frame members with a longitudinal axis, it is common to work with stress resultants in addition to the stresses themselves. For that reason, Figure 2 is included on the next page to show that section integration equations are then required in order to relate the stress resultant to the internal stress in the material. To be even more specific, Figure 3 displays the ingredients of the specific case of Euler-Bernoulli beam bending. This boundary value problem is addressed elsewhere on this website, working with distributed external load, q, bending moment, M, axial stress, σ, axial strain, ε, and lateral displacement w. The ingredients of a different and slightly more complex boundary value problem are shown in Figure 4; a 2D elasticity problem is here formulated in the x-y coordinate system with $u(x,y)$ and $v(x,y)$ being the displacements in the x and y directions, respectively, with $p_x(x,y)$ and $p_y(x,y)$ being the distributed external loads in those directions. In other documents it is shown that these boundary value problems can be formulated in a "stong form" (the differential equation); the "weighted residual form;" the "weak form" (the principle of virtual work); and the "variational form" (representing energy principles).

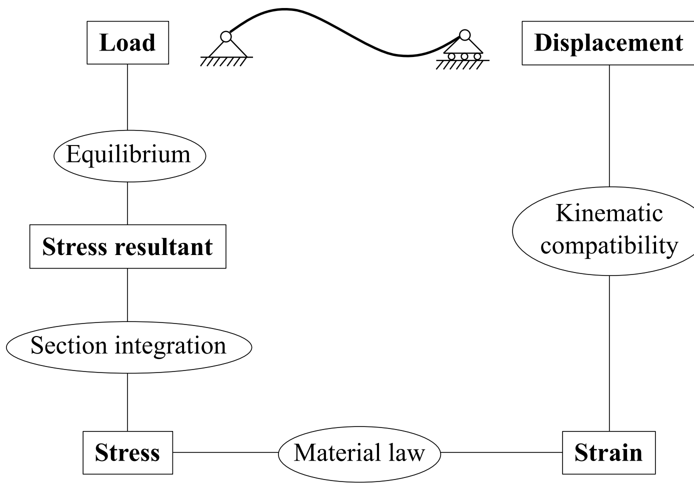

Figure 2: Ingredients of structural boundary value problems.

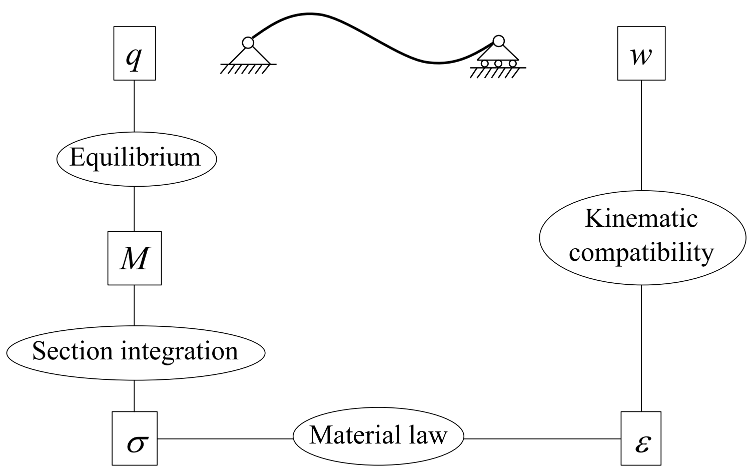

Figure 3: Ingredients of Euler-Bernoulli beam bending.

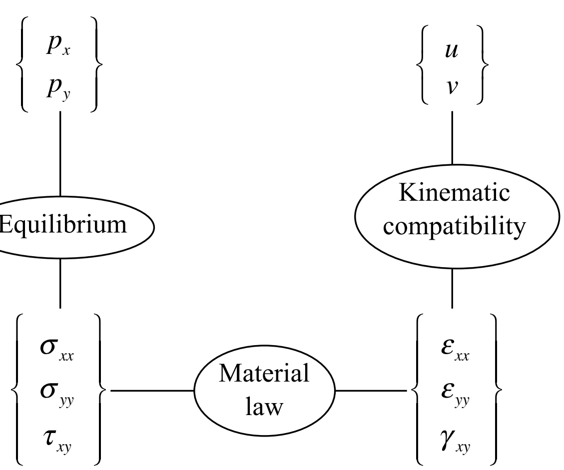

Figure 4: Ingredients of 2D boundary value problems.