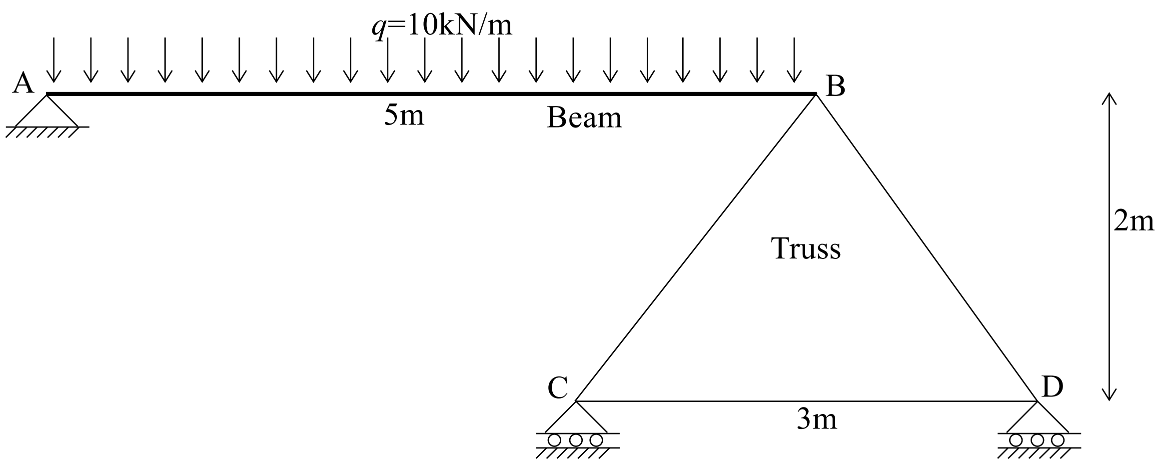
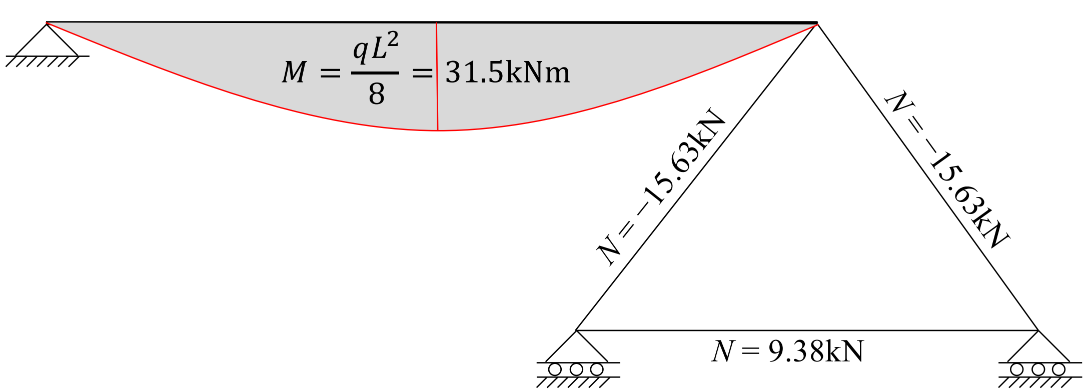
Beam Supported by Triangle Truss

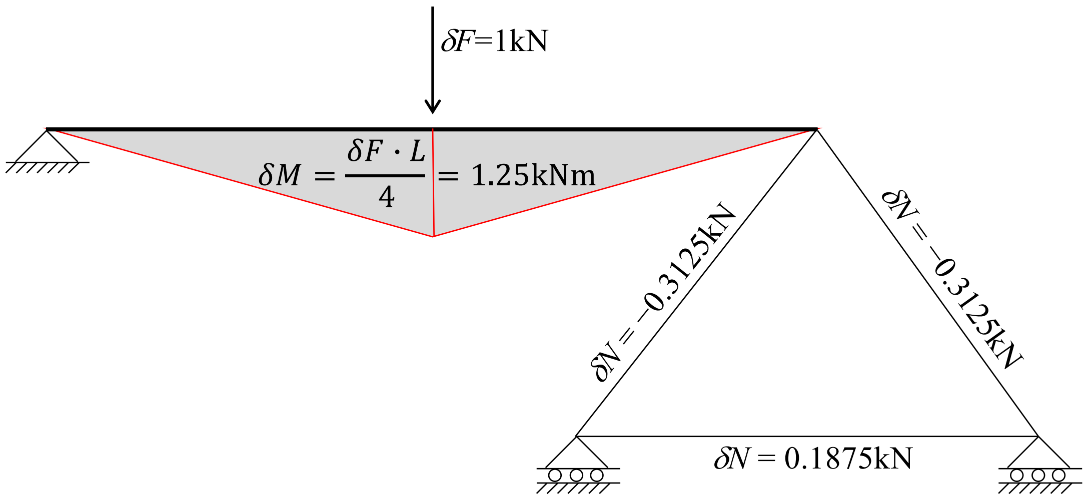
For the structure shown below, we wish to use virtual work, specifically the principle of virtual forces, in order to determine the vertical displacement at the middle of the beam that spans between A and B. At point B the beam rests on a truss structure, and this example helps show the "magic" of virtual work calculations. Specifically, we do NOT need to find the vertical displacement at B in order to find the midspan beam displacement. We simply place a unit load where we seek the displacement, and all contributions to that displacement are automatically included.



We start by determining the internal forces due to the externally applied real load, q, for this statically determinate structure by equilibrium:



Next, we place a unit virtual load at the location where we wish to determine the displacement, and again apply equilibrium in order to determine the virtual internal forces:



The sought displacement is now calculated by adding up virtual work, i.e., "virtual internal force times real internal deformation." Notice the use of quick-integration formulas for the integration of virtual moment with real curvature:

$$\delta F \cdot \Delta = \int \delta M \cdot \frac{M}{EI}\, dx + \sum \delta N \cdot \frac{N}{EA} \cdot L$$

$$+\frac{5}{12EI} \cdot 1.25\text{kNm} \cdot 31.5\text{kNm} \cdot 5\text{m}$$

$$+2 \cdot (-0.3125\text{kN}) \cdot \frac{-15.63\text{kN}}{EA} \cdot \sqrt{(3\text{m})^2 + (1.5\text{m})^2}$$

$$+0.1875\text{kN} \cdot \frac{9.38\text{kN}}{EA} \cdot 3\text{m}$$

$$= \frac{82.03}{EI} + \frac{38.04}{EA}$$

By using the values EI=20,000kNm^2 and EA=375,000kN we find that the contribution to the midspan beam deflection from axial deformation in the truss members is significantly smaller than the flexural deformation of the beam:

$$\Delta = \frac{82.03}{EI} + \frac{38.04}{EA} = 0.0041\text{m} + 0.0001\text{m} = 0.0042\text{m}$$