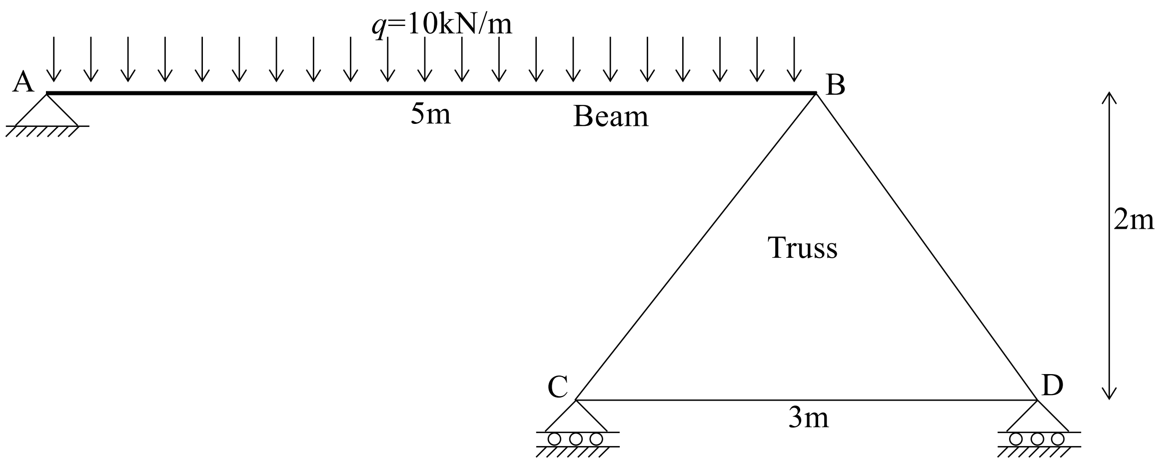
Beam Supported by Triangle Truss

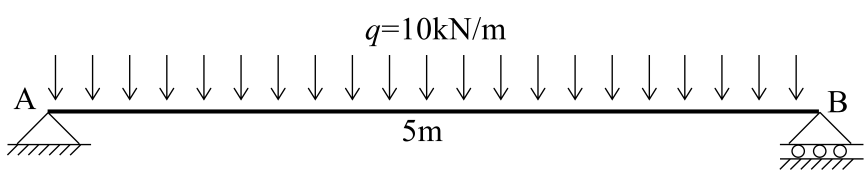
Consider the mixed beam-truss structure shown below. According to the formula used on this website, the structure is statically determinate:

$$DSI = (f \cdot m + s) - (e \cdot j + h) = (3 \cdot 1 + 1 \cdot 3 + 4) - (3 \cdot 2 + 2 \cdot 2 + 0) = 0$$

The objective is to determine the internal forces in the structure, i.e., the bending moment diagram (BMD) and shear force diagram (SFD) in the beam and the axial forces in the truss members.



Because the structure is statically determinate, deformations do not affect the internal forces. That means the beam feels simply supported, regardless of the deflection at B. As a result, we start by analyzing the following simply supported beam:



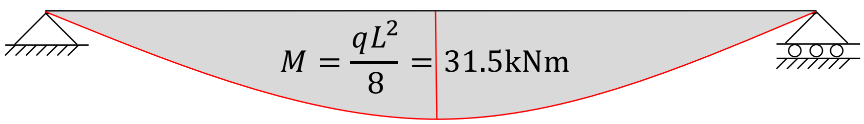
The sum of moments about A gives the reaction force at B, which is $qL/2$, as expected:

$$\Sigma M_A = 10\text{kN/m} \cdot 5\text{m} \cdot 2.5\text{m} - R_{By} \cdot 5\text{m} = 0 \rightarrow R_{By} = 25\text{kN}$$

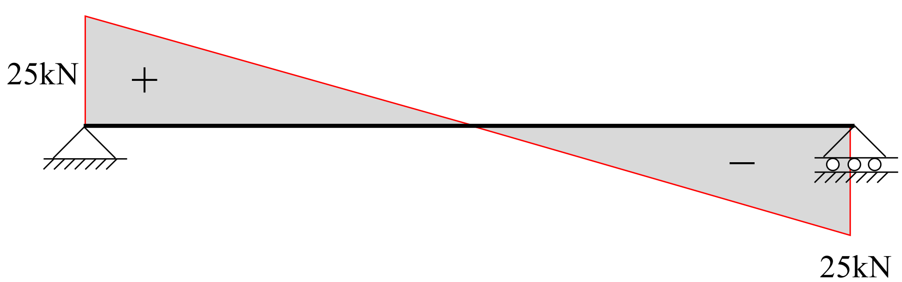
As a result, the moment at midspan is $qL^2/8$, as expected, with tension at the bottom:

$$M = 25\text{kN} \cdot 2.5\text{m} - 10\text{kN/m} \cdot 2.5\text{m} \cdot 1.25\text{m} = 31.25\text{kNm}$$

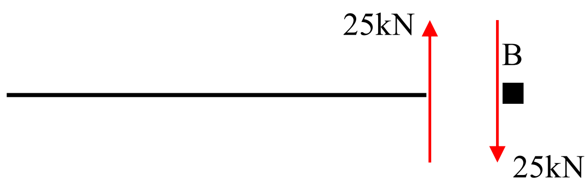
The load is uniform, meaning the shear force diagram is linear, and the bending moment diagram is a parabola:



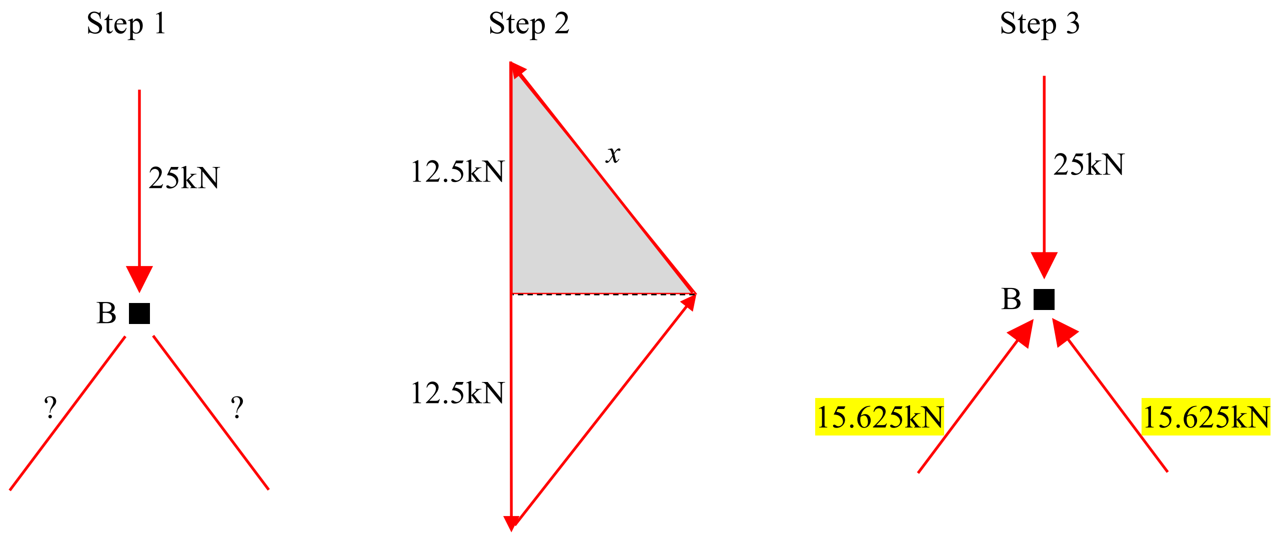
By sum of forces in the vertical direction, the vertical reaction force at A is the same as at B. In other words, the shear force at both ends is $qL/2$ = 25kN. That means the shear force diagram, formally $V=dM/dx$ with "downhill walk on the moment diagram giving positive clockwise shear," is:



The shear force at B transfers to the truss structure below that point. Joint equilibrium at B starts with the transfer of the shear force in the beam onto the joint:

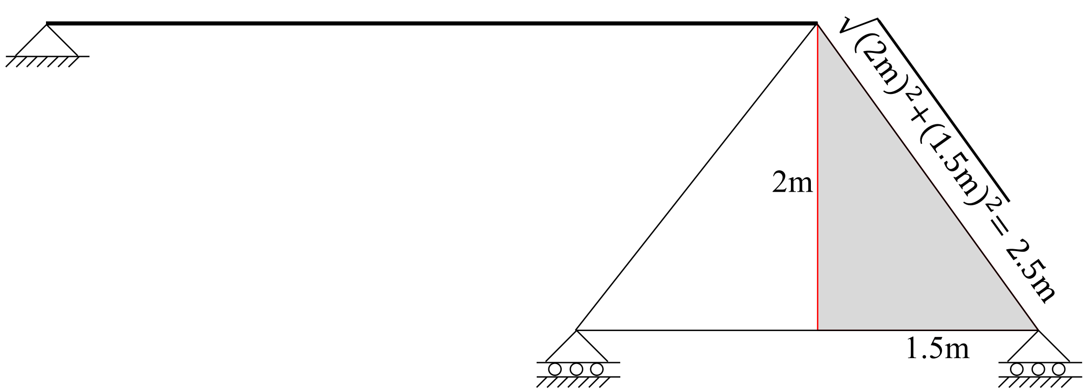


Next, joint equilibrium gives the axial force in two of the truss members:

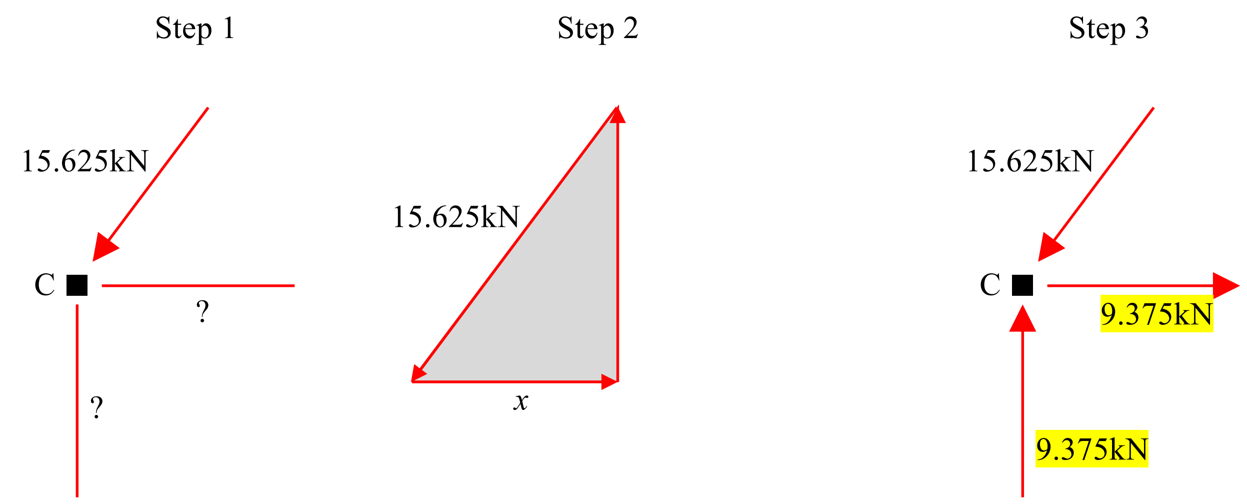


Similarity between the shaded triangle in the force polygon above and the shaded triangle below gives the axial force in the inclined truss members:

$$\frac{x}{12.5\text{kN}} = \frac{2.5\text{m}}{2\text{m}} \quad \rightarrow \quad x = \frac{2.5\text{m}}{2\text{m}} \cdot 12.5\text{kN} = 15.625\text{kN}$$

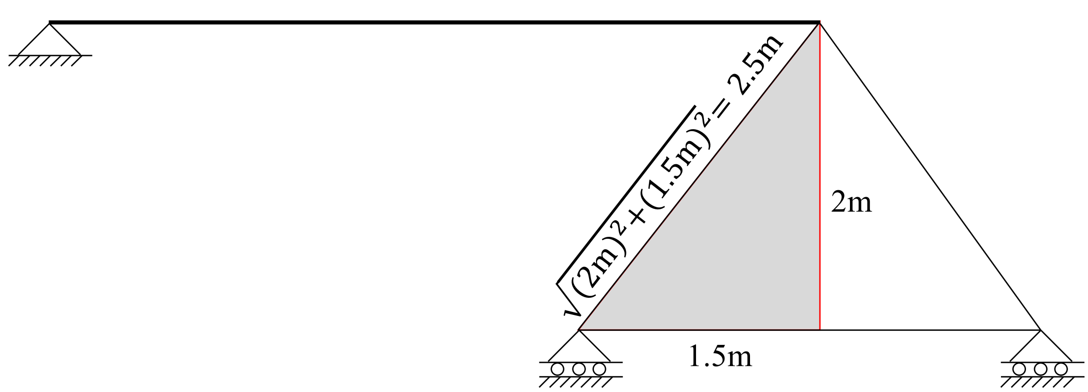


Joint equilibrium at C gives the last unknown truss force:



Similarity between the shaded triangle in the force polygon above and the shaded triangle below gives the axial force in the horizontal truss member:

$$\frac{x}{15.625\text{kN}} = \frac{1.5\text{m}}{2.5\text{m}} \quad \rightarrow \quad x = \frac{1.5\text{m}}{2.5\text{m}} \cdot 15.625\text{kN} = 9.375\text{kN}$$



A summary of the primary internal forces to cause deformation in the structure, i.e., bending moment and axial force, is provided here: