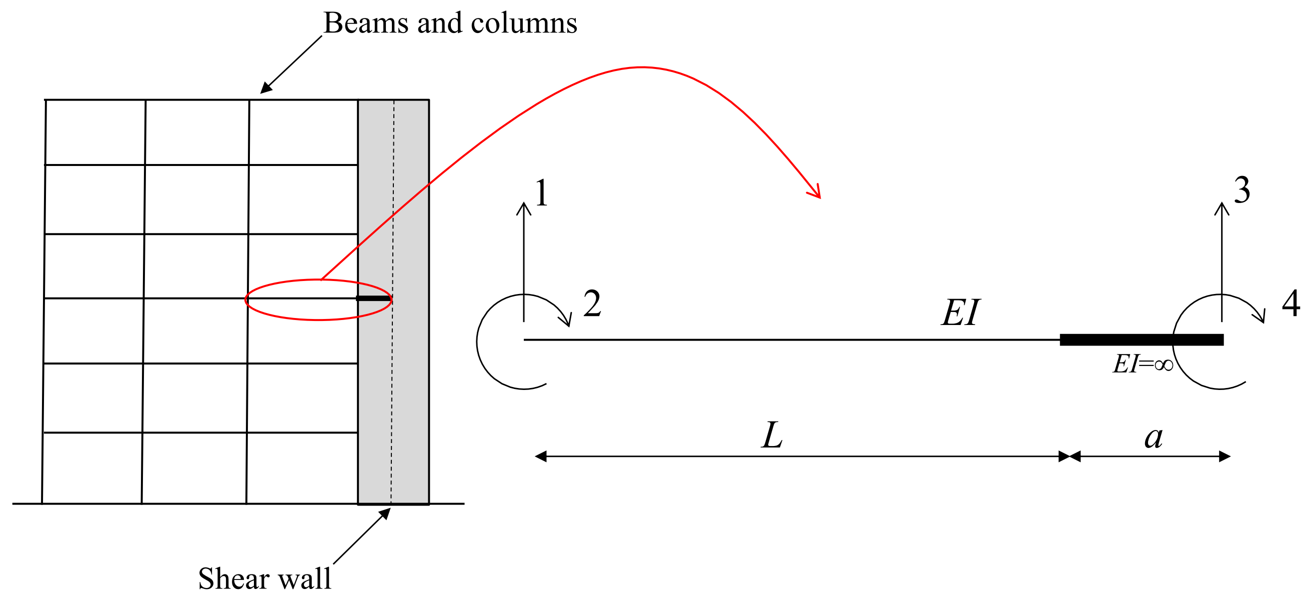
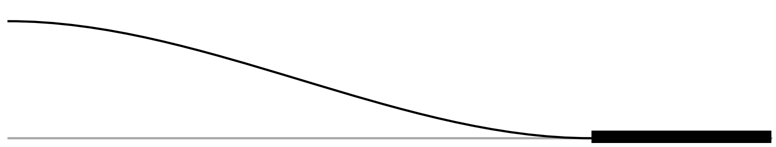
Beam with Rigid Part

Consider the beam element shown below. It has four degrees of freedom and an infinitely rigid part, marked with a thick black line. The element is intended to model the element circled in the figure on the left-hand side. It is understood that the infinitely rigid part is intended to model the shear wall, which is much stiffer than the elastic beam element, which has bending stiffness denoted *EI*.



Stiffness matrix by the classical stiffness method

The **first column** of the stiffness matrix is established by setting the first degree of freedom equal to one, while keep all the others at zero:

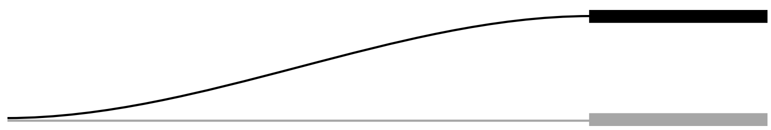


$$\text{column1} = \left\{\frac{12\,EI}{L^3},\ -\frac{6\,EI}{L^2},\ -\frac{12\,EI}{L^3},\ -\frac{6\,EI}{L^2} - \frac{12\,EI}{L^3}\,a\right\};$$

Second column:

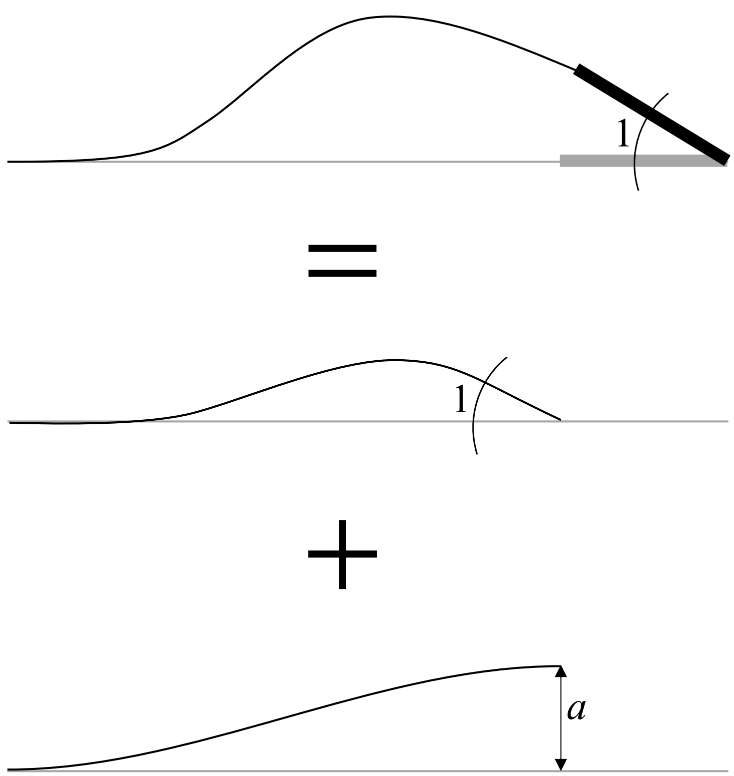
$$\text{column2} = \left\{-\frac{6\,EI}{L^2},\ \frac{4\,EI}{L},\ \frac{6\,EI}{L^2},\ \frac{2\,EI}{L} + \frac{6\,EI}{L^2}\,a\right\};$$

Third column:



$$\text{column3} = \left\{-\frac{12\,EI}{L^3},\ \frac{6\,EI}{L^2},\ \frac{12\,EI}{L^3},\ \frac{6\,EI}{L^2} + a\,\frac{12\,EI}{L^3}\right\};$$

Fourth column:



$$\text{column4} = \left\{-\frac{6\,EI}{L^2} - \frac{12\,EI}{L^3}\,a,\ \frac{2\,EI}{L} + \frac{6\,EI}{L^2}\,a,\ \frac{6\,EI}{L^2} + \frac{12\,EI}{L^3}\,a,\ \left(\frac{4\,EI}{L} + \frac{6\,EI}{L^2}\,a\right) + \left(\frac{6\,EI}{L^2}\,a + \frac{12\,EI}{L^3}\,a^2\right)\right\};$$

In summary, that stiffness matrix is:

```
KfinalManual = {column1, column2, column3, column4};
KfinalManual // MatrixForm
```

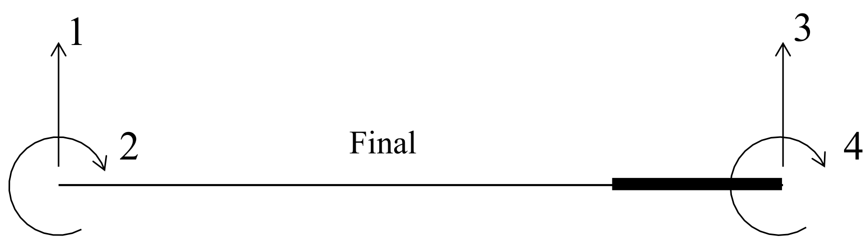
which yields:
$$\begin{pmatrix} \frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} & -\frac{12\,EI}{L^3} & -\frac{12\,a\,EI}{L^3}-\frac{6\,EI}{L^2} \\ -\frac{6\,EI}{L^2} & \frac{4\,EI}{L} & \frac{6\,EI}{L^2} & \frac{6\,a\,EI}{L^2}+\frac{2\,EI}{L} \\ -\frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & \frac{12\,EI}{L^3} & \frac{12\,a\,EI}{L^3}+\frac{6\,EI}{L^2} \\ -\frac{12\,a\,EI}{L^3}-\frac{6\,EI}{L^2} & \frac{6\,a\,EI}{L^2}+\frac{2\,EI}{L} & \frac{12\,a\,EI}{L^3}+\frac{6\,EI}{L^2} & \frac{12\,a^2\,EI}{L^3}+\frac{12\,a\,EI}{L^2}+\frac{4\,EI}{L} \end{pmatrix}$$

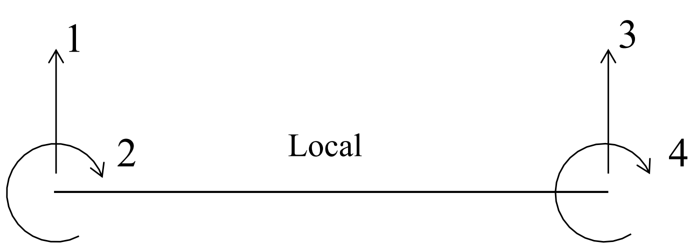
Check that the stiffness matrix is symmetric:

```
SymmetricMatrixQ[KfinalManual]
```

which yields: `True`

Stiffness matrix by the computational stiffness method, i.e., using a transformation matrix





The transformation matrix $\mathbf{T}_{lf}$ is obtained by setting the DOFs of the Final configuration equl to one, one at a time, which forms the columns of $\mathbf{T}_{lf}$:

```
Tlf = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, a}, {0, 0, 0, 1}};
Tlf // MatrixForm
```

which yields: $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a \\ 0 & 0 & 0 & 1 \end{pmatrix}$

The stiffness matrix in the local configuration is:

```
Kl = {{12 EI/L^3, -6 EI/L^2, -12 EI/L^3, -6 EI/L^2}, {-6 EI/L^2, 4 EI/L, 6 EI/L^2, 2 EI/L},
   {-12 EI/L^3, 6 EI/L^2, 12 EI/L^3, 6 EI/L^2}, {-6 EI/L^2, 2 EI/L, 6 EI/L^2, 4 EI/L}};
Kl // MatrixForm
```

which yields: $\begin{pmatrix} \frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} & -\frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} \\ -\frac{6\,EI}{L^2} & \frac{4\,EI}{L} & \frac{6\,EI}{L^2} & \frac{2\,EI}{L} \\ -\frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & \frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} \\ -\frac{6\,EI}{L^2} & \frac{2\,EI}{L} & \frac{6\,EI}{L^2} & \frac{4\,EI}{L} \end{pmatrix}$

That means that the final stiffness matrix, using this approach, comes out the same as in a):

```
KfinalTransform = Transpose[Tlf].Kl.Tlf;
KfinalTransform // MatrixForm
```

which yields: $\begin{pmatrix} \frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} & -\frac{12\,EI}{L^3} & -\frac{12\,a\,EI}{L^3} - \frac{6\,EI}{L^2} \\ -\frac{6\,EI}{L^2} & \frac{4\,EI}{L} & \frac{6\,EI}{L^2} & \frac{6\,a\,EI}{L^2} + \frac{2\,EI}{L} \\ -\frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & \frac{12\,EI}{L^3} & \frac{12\,a\,EI}{L^3} + \frac{6\,EI}{L^2} \\ -\frac{12\,a\,EI}{L^3} - \frac{6\,EI}{L^2} & \frac{6\,a\,EI}{L^2} + \frac{2\,EI}{L} & \frac{12\,a\,EI}{L^3} + \frac{6\,EI}{L^2} & a\left(\frac{12\,a\,EI}{L^3} + \frac{6\,EI}{L^2}\right) + \frac{6\,a\,EI}{L^2} + \frac{4\,EI}{L} \end{pmatrix}$

Compare with the result from a):

```
difference = KfinalManual - KfinalTransform // Simplify // MatrixForm
```

which yields: $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$