A short course on

Indeterminate Structures

This video:

Moment Distribution Method

Terje's Toolbox is freely available at terje.civil.ubc.ca
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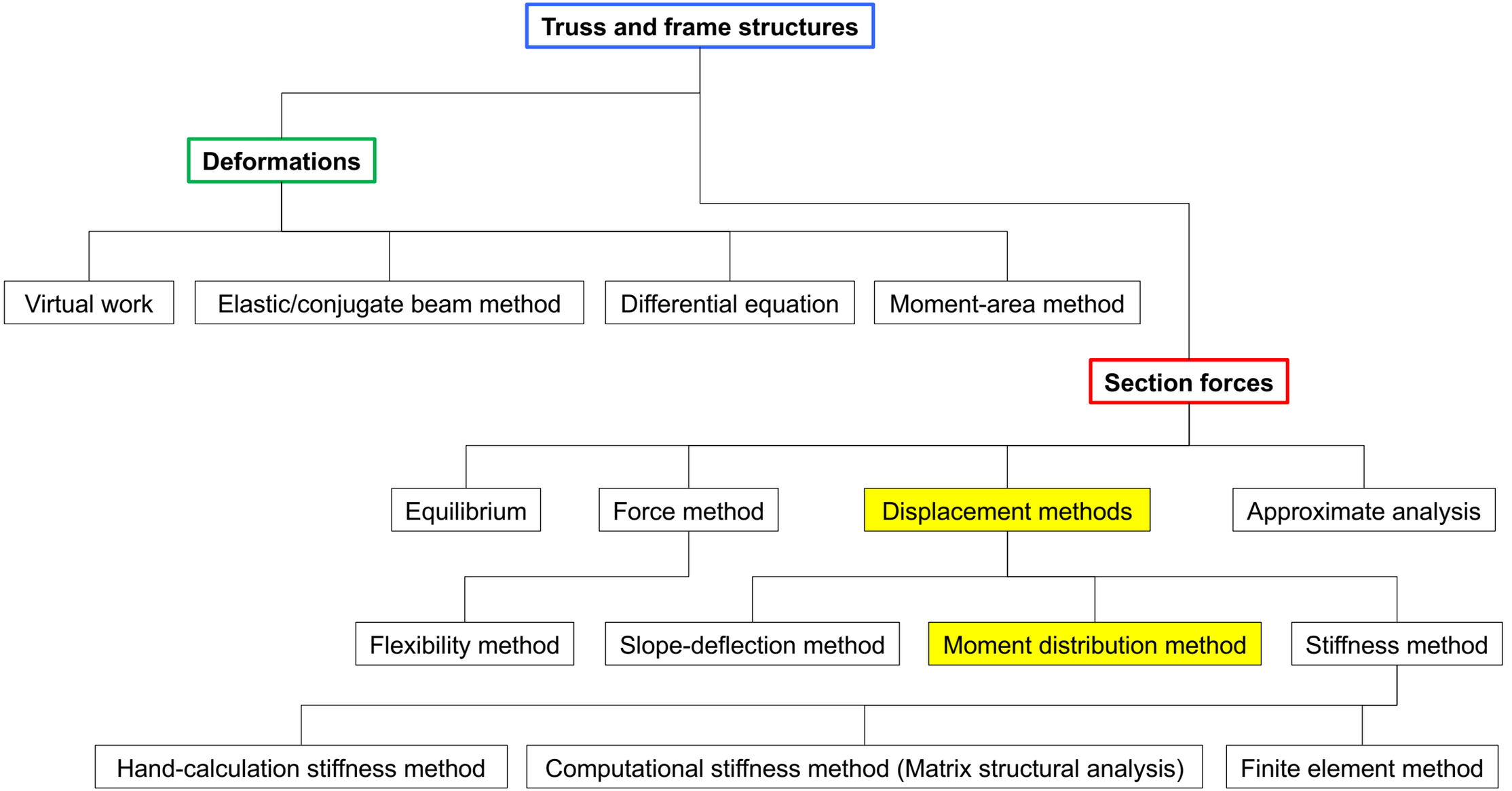
Overview of Methods



Clamp & Release

All displacement methods:

Establish equilibrium = **CLAMP**

Solve = **RELEASE**

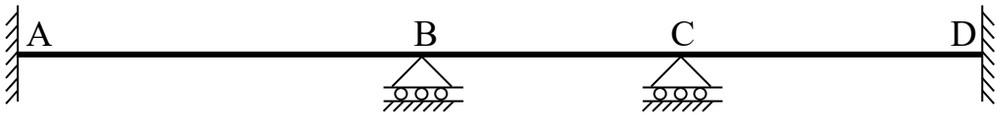
Moment Distribution

Hardy Cross (1930)

Sometimes called the Cross Method

Unclamp one joint at a time

Format



DF			
FEM			
DEM			
COM			
DEM			
COM			
DEM			
COM			
DEM			
COM			
SUM	... ...	... ...	... ...

DF = distribution factor

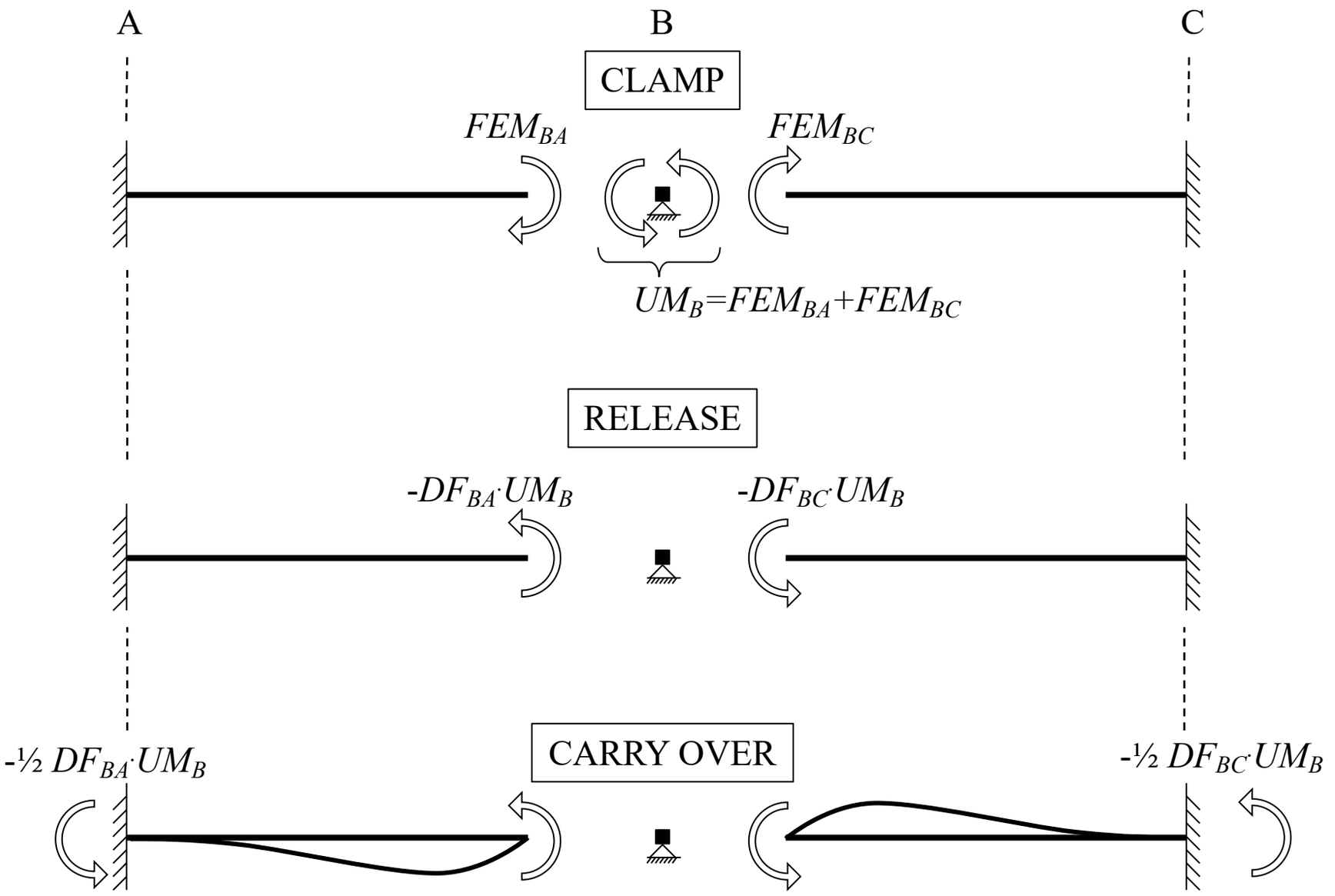
FEM = fixed-end moment

DEM = distributed end moment

COM = carry-over moment

SUM = sum that gives final end moments

Derivation



Unbalanced moment at B:

$$UM_B = FEM_{BA} + FEM_{BC}$$

Distribution Factors

$$\theta_{BA} = \frac{-\left(DF_{BA} \cdot UM_B\right) \cdot L_{AB}}{4EI_{AB}}$$

$$\theta_{BC} = \frac{-\left(DF_{BC} \cdot UM_B\right) \cdot L_{BC}}{4EI_{BC}}$$

$$DF_{BA} = \frac{4EI_{AB}/L_{AB}}{4EI_{AB}/L_{AB} + 4EI_{BC}/L_{BC}}$$

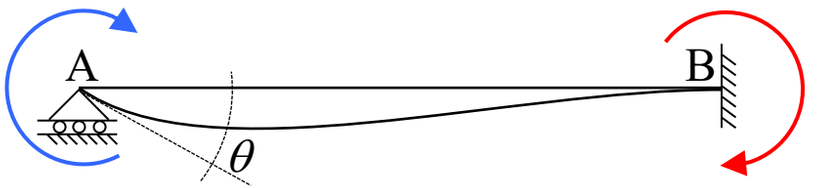
$$DF_{BC} = \frac{4EI_{BC}/L_{BC}}{4EI_{AB}/L_{AB} + 4EI_{BC}/L_{BC}}$$

$$DF_i = \frac{4EI_i / L_i}{\sum 4EI / L}$$

Carry-over Moments

Use the slope deflection equation:

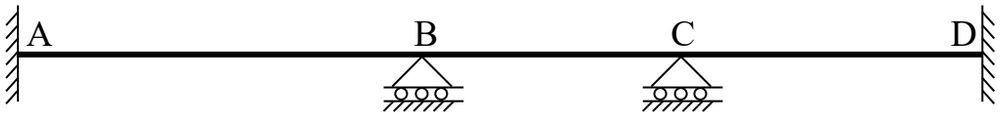
$$M_{AB} = \frac{2EI}{L}(2\theta) = \frac{\boldsymbol{4EI}}{\boldsymbol{L}}(\theta)$$



$$M_{BA} = \frac{\boldsymbol{2EI}}{\boldsymbol{L}}(\theta)$$

Result: $COM_{BA} = \frac{1}{2} \cdot DF_{BA} \cdot UM_{B}$

Procedure



DF			✓	✓	✓	✓				
FEM	✓		✓	✓	✓	✓			✓	
DEM			✓	✓						
COM	✓				✓					
DEM					✓	✓				
COM				✓					✓	
DEM			✓	✓						
COM	✓				✓					
DEM					✓	✓				
COM				✓					✓	
	…		…	…	…	…	…		…	
SUM	✓		✓	✓	✓		✓		✓	

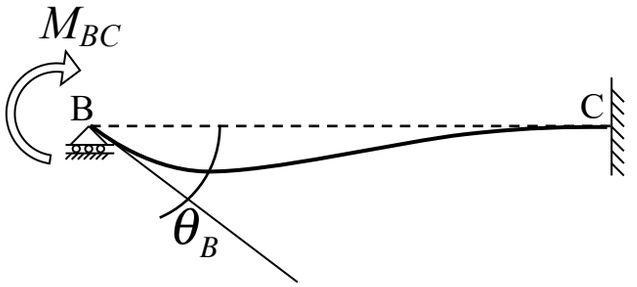
Modified Distribution Factors

Default: $$DF_i = \frac{4EI_i / L_i}{\sum 4EI / L}$$

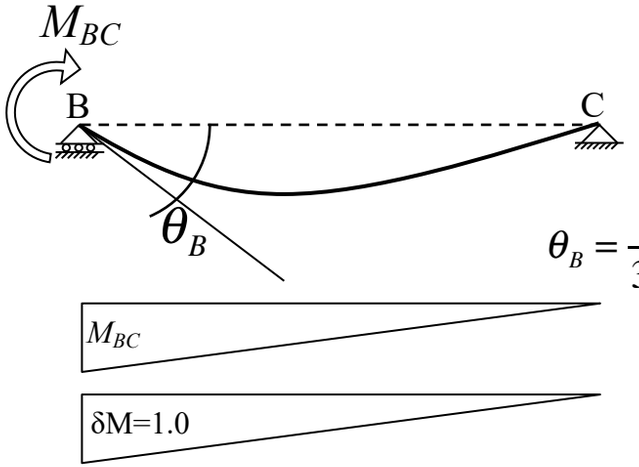
What if the other end is a **pin or roller at the end**?

What if the other end is across a **symmetry** line?

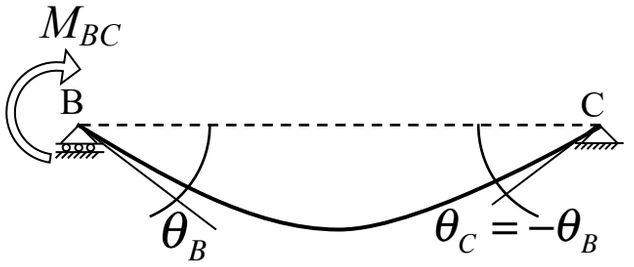
Derivations



$$M_{BC} = \frac{2EI}{L}\left(2\cdot\theta_B\right) = \frac{4EI}{L}\cdot\theta_B$$



$$\theta_B = \frac{1}{3EI}\cdot(1)\cdot\left(M_{BC}\right)\cdot L \Rightarrow M_{BC} = \frac{3EI}{L}\cdot\theta_B$$



$$M_{BC} = \frac{2EI}{L}\left(2\cdot\theta_B - \theta_B\right) = \frac{2EI}{L}\cdot\theta_B$$

More lectures:

Terje's Toobox:

terje.civil.ubc.ca