Poisson occurrences

Wind and snow

Many codes suggest designing against "1 in 50" loads

a) What is the annual probability of exceedance of the specified loads?

b) What is the return period and rate of exceedance of the specified loads?

c) What does the return period represent?

d) What is the probability that the specified loads will be exceeded in 10 years?

Earthquakes

The building code suggests designing against "2% in 50" earthquakes

a) What is the return period and rate of exceedance of that intensity level?

b) What is the probability of exceedance of that intensity level in any given year?

c) What is the probability of exceedance of that intensity level in 10 years?

d) If it becomes known that an earthquake occurred in 2015 that exceeded the design-intensity, what is the probability of exceedance of that intensity level in 2016?

Storms and floods

Some guidelines suggests design against the "100-year storm" or the "100-year flood"

a) What is the chance of such a storm/flood in any given year?

b) What is the chance of such a storm/flood in 10 years?

c) What is the chance of such a storm/flood in 100 years?

Northern Gateway

It has been suggested that oil spills might occur with a return period of 550 years

a) What is the chance of an oil spill in 10 years?

b) What is the chance of an oil spill in 50 years?

Combined hazards

Assuming independent processes

a) What is the probability that we will have BOTH an earthquake and an oil spill in the next 50 years?

b) What is the probability that we will have EITHER an earthquake or an oil spill in the next 50 years BUT NOT BOTH?

c) What is the probability that we will have EITHER an earthquake or an oil spill in the next 50 years OR BOTH?

d) If the failure probability is 0.004 due to snow and 0.1 due to earthquakes, what is the failure probability in the next 50 years?

Anchoring to annual occurrence probability
a) Symbolically, determine the probability of occurrence in 75 years from the annual occurrence probability WITHOUT determining λ
b) Symbolically, determine the probability of occurrence in 75 years from the annual occurrence probability by considering each year as a trial of a Bernoulli sequence

Wind and snow

It is actually the annual probability that is given, which for small probabilities is close to the annual rate:

$$\text{pAnnualSnow} = \frac{1}{50} \;//\; \text{N}$$

which yields: 0.02

Corresponding rate:

$$\lambda\text{snow} = -\text{Log}[1 - \text{pAnnualSnow}]$$

which yields: 0.0202027

Corresponding return period:

$$\text{Rsnow} = \frac{1}{\lambda\text{snow}}$$

which yields: 49.4983

Probability of occurrence in 10 year:

$$1 - \text{Exp}[-\lambda\text{snow}\ 10]$$

which yields: 0.182927

Out of curiousity, probability of occurrence in 50 year:

$$1 - \text{Exp}[-\lambda\text{snow}\ 50]$$

which yields: 0.63583

Earthquakes

Here it is the probability of occurrence in 50 years that is given:

```
pFiftyEq = 0.02;
```

Solve for the rate:

$$\lambda eq = -\frac{\text{Log}[1 - \text{pFiftyEq}]}{50}$$

which yields: 0.000404054

Corresponding return period:

$$\text{Req} = \frac{1}{\lambda eq}$$

which yields: 2474.92

Annual occurrence probability:

```
1 - Exp[-λeq 1]
```

which yields: 0.000403973

Occurrence probability in 10 years:

```
1 - Exp[-λeq 10]
```

which yields: 0.00403239

Storms and floods

Here it is the return period that is given:

```
Rstorm = 100;
```

Annual probability of occurrence:

$$1 - \text{Exp}\left[-\frac{1}{\text{Rstorm}}\right] \; // \; \text{N}$$

which yields: 0.00995017

Probability of occurrence in 10 years:

```
1 - Exp[-1/Rstorm 10] // N
```

which yields: 0.0951626

Probability of occurrence in 100 years:

```
1 - Exp[-1/Rstorm 100] // N
```

which yields: 0.632121

Tanker oil spills

Here it is again the return period that is given

```
Roil = 550;
```

Probability of occurrence in 10 years:

```
1 - Exp[-1/Roil 10] // N
```

which yields: 0.0180175

Probability of occurrence in 50 years:

```
1 - Exp[-1/Roil 50] // N
```

which yields: 0.0868993

Combined hazards

Probability of both

Individual probabilities:

```
Peq = 1 - Exp[-λeq 50] // N
```

which yields: 0.02

$$\text{Poil} = 1 - \text{Exp}\left[-\frac{1}{\text{Roil}}\,50\right] \,//\, N$$

which yields: 0.0868993

Assuming independence, the probability of both occurring:

```
Peq Poil
```

which yields: 0.00173799

Probability of either or both

Assuming independence, the union rule gives the following probability for oil spill or earthquake or both:

```
Peq + Poil - Peq Poil
```

which yields: 0.105161

This answer can also be obtained by first combining the processes into one by adding the rates:

```
λcomb = λoil + λeq
```

which yields: 0.000404054 + λoil

... and then compute the probability of occurrence:

```
1 - Exp[-λcomb 50]
```

which yields: $1 - e^{50\times(-0.000404054-\lambda\text{oil})}$

Probability of either but not both

This is the union rule, as above, but with the intersection term subtracted once more:

```
Peq + Poil - Peq Poil - Peq Poil
```

which yields: 0.103423

Failure probability

The Poisson process for failure due to snow has rate

```
λsnowFail = λsnow 0.004;
```

The Poisson process for failure due to earthquakes has rate:

```
λeqFail = λeq 0.1;
```

The rate of failure considering both processes is:

```
λFail = λsnowFail + λeqFail
```

which yields: 0.000121216

The probability of occurrence for this failure process in 50 years is:

```
1 - Exp[-λFail 50]
```

which yields: 0.00604248

Anchoring to annual occurrence probability

Using the Poisson distribution to find an occurrence probability essentially means, in this case: $p_1 = 1 - e^{-\lambda}$ and $p_{75} = 1 - e^{-75\lambda}$. The first of those expressions is rearranged to read $e^{-\lambda} = 1 - p_1$ and the second expression is written $p_{75} = 1 - \left(e^{-\lambda}\right)^{75}$. Substituting the first into the second yields $p_{75} = 1 - \left(e^{-\lambda}\right)^{75} = 1 - (1 - p_1)^{75}$.

The probability of occurrence, i.e., "success" in a Bernoulli sequence is $1 - p(x) = (n \mid x)\,(p)^x\,(1-p)^{n-x}$, where the binomial coefficient has been written $(n \mid x)$, for convenience. The following probability emerges for $x = 0$ and $n = 75$: $1 - p(0) = 1 - 1\,(p)^0\,(1-p)^{75-0} = 1 - (1-p)^{75}$.