A short course on

Indeterminate Structures

This video:

Stiffness Method

Terje's Toolbox is freely available at terje.civil.ubc.ca
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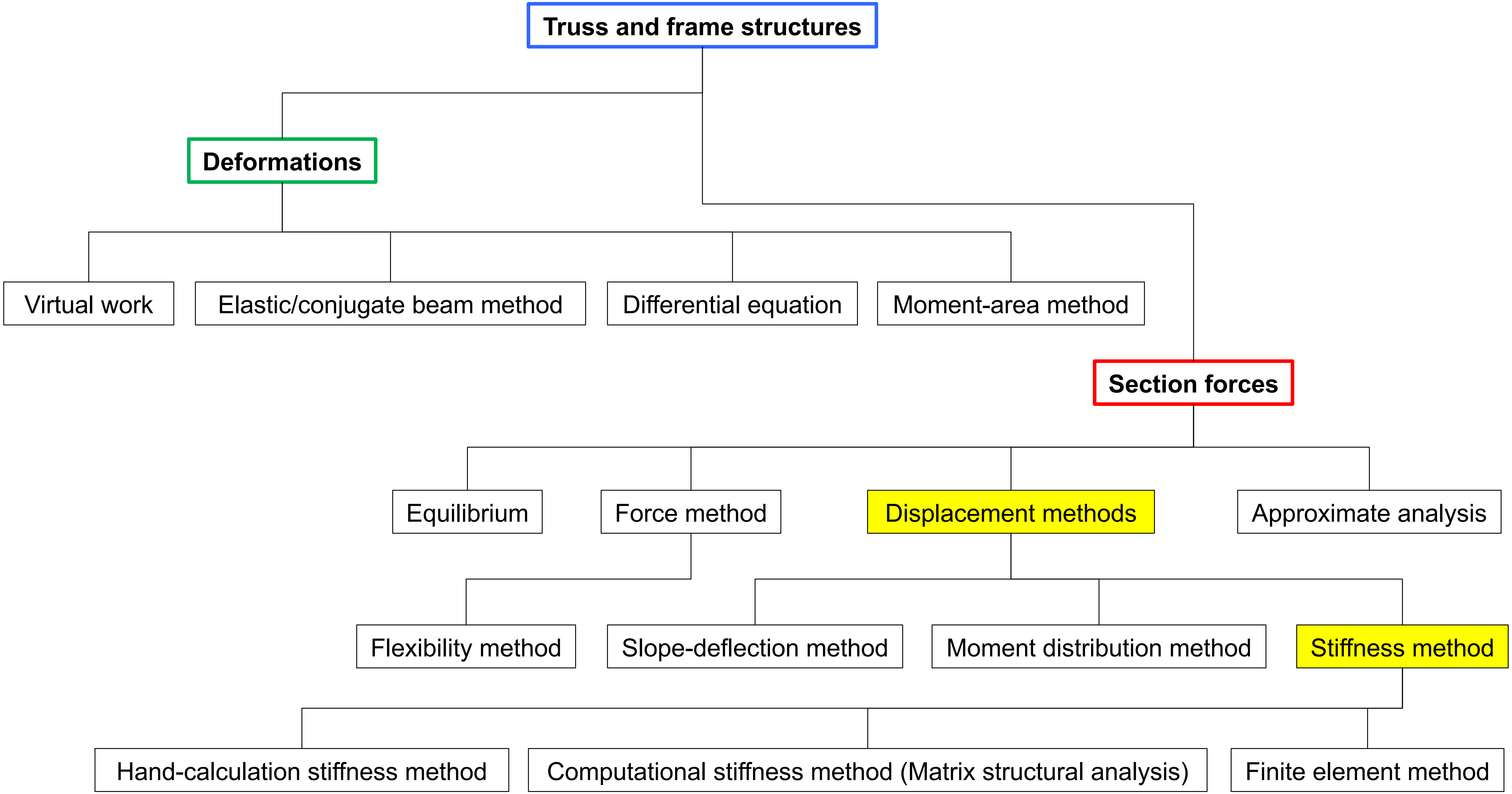
Overview of Methods



Reference: Slope Deflection Method

1. Identify the degrees of freedom, i.e., the unknown displacements and rotations
2. Establish equilibrium equations in terms of end moments
3. Substitute slope-deflection equation for end moments
4. Solve for the unknown displacements and rotations
5. Substitute displacements and rotations into slope-deflection equation to get end moments
6. Draw bending moment diagram between known end moment values

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$$\left(\quad \right)\theta_A + \left(\quad \right)\theta_B = \text{loads}$$

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Stiffness Method

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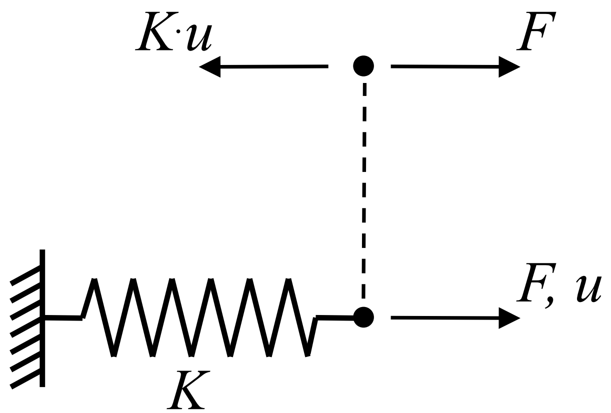
$\Rightarrow$

$$\mathbf{Ku} = \mathbf{F}$$

Stiffness Method Procedure

1. Identify the degrees of freedom, i.e., the unknown displacements and rotations
2. Establish stiffness matrix, **K**
3. Establish load vector, **F**
4. Solve for the unknown displacements and rotations
5. Substitute displacements and rotations into slope-deflection equation to get end moments
6. Draw bending moment diagram between known end moment values

The Stiffness Concept



Equilibrium: $K\,u = F$

More Degrees of Freedom

$$K_{11}\,u_1 + K_{12}\,u_2 = F_1$$
$$K_{21}\,u_1 + K_{22}\,u_2 = F_2$$

$$\mathbf{K}\,\mathbf{u} = \mathbf{F}$$

$$K_{ij}\,u_j = F_i$$

K_{ij} = force along DOF number *i* due to a unit displacement or rotation along DOF number *j*
u_j = unknown displacement or rotation along DOF number *j*
F_i = force along DOF number *i* due to external loads

Establish K

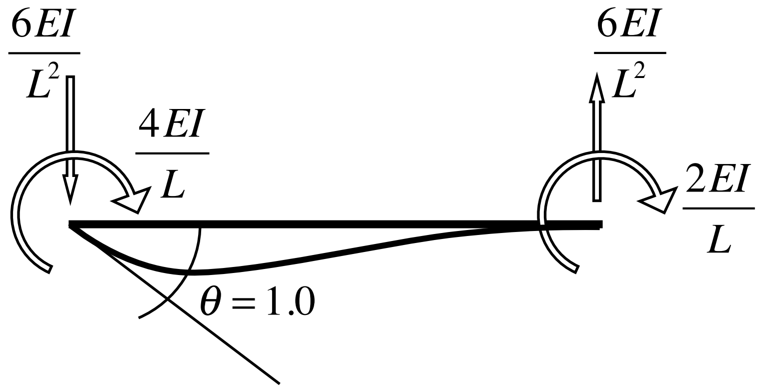
1. Sketch the displaced shape of the structure for a unit displacement or rotation along DOF number *j*, with all other DOFs clamped

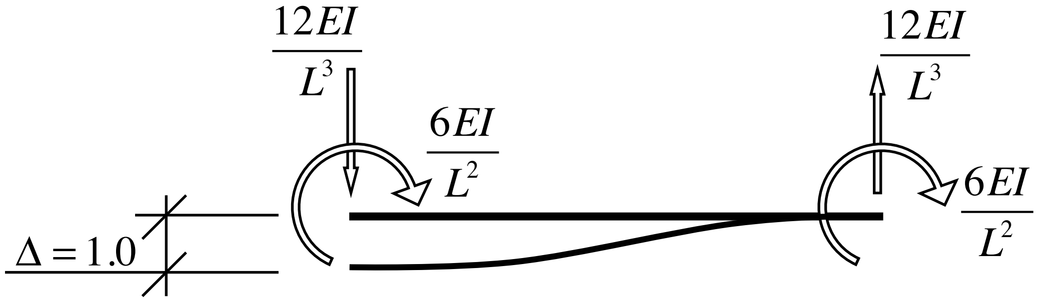
2. Determine the force along every DOF to maintain this displaced shape, i.e., K_{ij}, which form column number *j* of the stiffness matrix

3. Carry out Step 1 and 2 for all DOFs to establish column by column of the stiffness matrix

4. Check that the **K** is symmetric with only positive entries on the diagonal

Formula Sheet





Load Vector

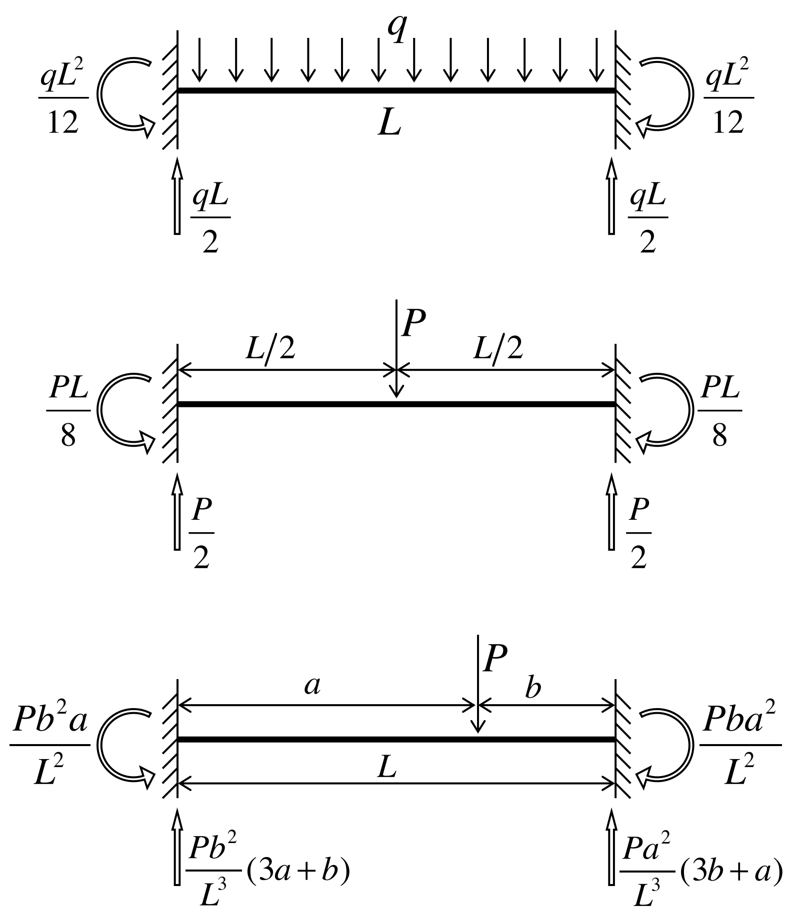
External forces applied to structure: $\mathbf{Ku} = \mathbf{F}$

Split member forces and point loads: $\mathbf{Ku} + \bar{\mathbf{F}} = \grave{\mathbf{F}}$

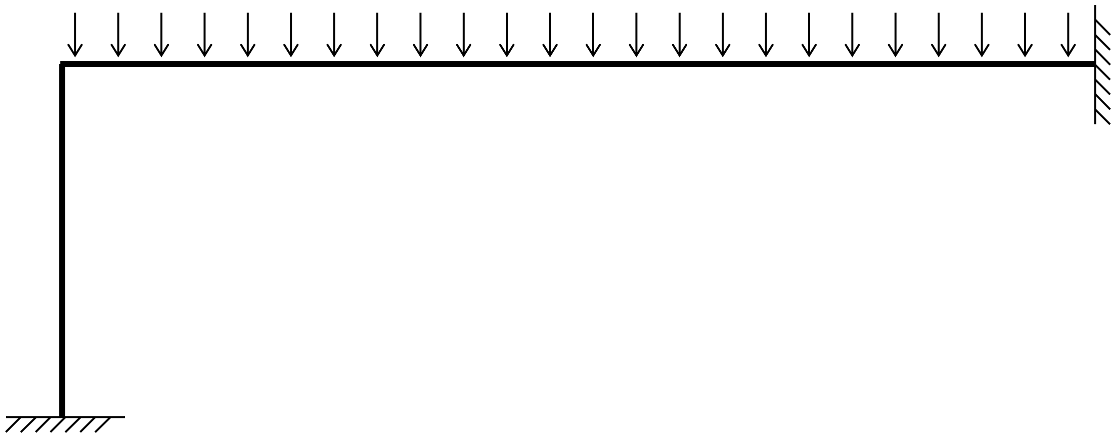
Total load vector: $\mathbf{Ku} = \grave{\mathbf{F}} - \bar{\mathbf{F}} = \mathbf{F}$

Member end forces after solving equilibrium equations: $\mathbf{F} = \mathbf{Ku} + \bar{\mathbf{F}}$

Formula Sheet for $\bar{\mathbf{F}}$



Example



Example

More lectures:

Terje’s Toobox:

terje.civil.ubc.ca