A short course on

Indeterminate Structures

This video:

Slope Deflection Method

Terje's Toolbox is freely available at terje.civil.ubc.ca
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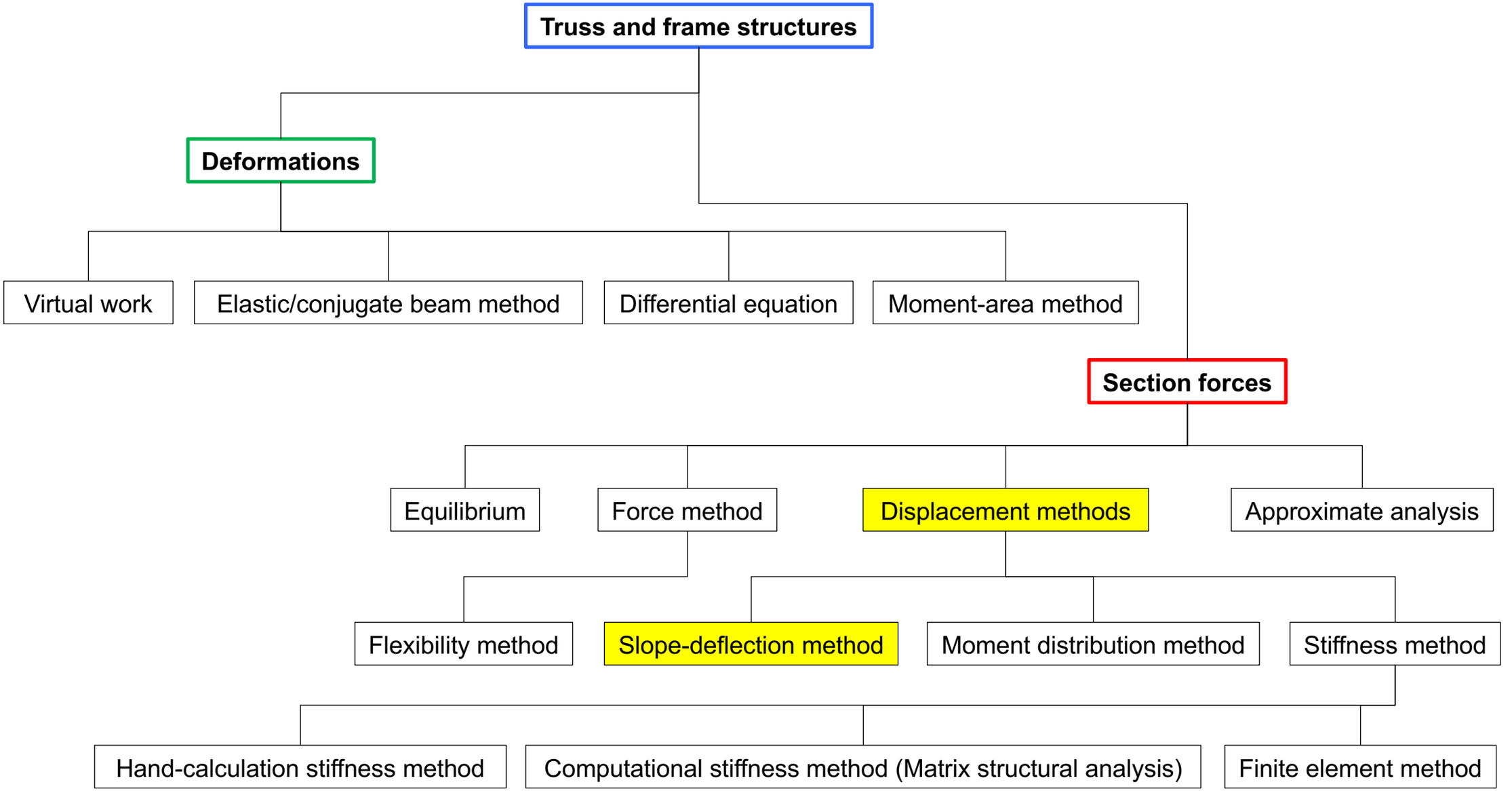
Overview of Methods



Two Worlds

Displacement methods

Degrees of freedom (displacements and rotations) are considered unknown

Impose **equilibrium** in order to solve for the unknowns

Force methods

As many redundant forces as the degree of static indeterminacy are considered unknown

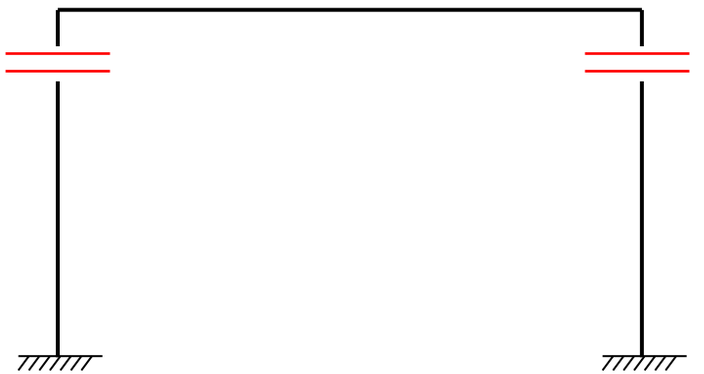
Impose kinematic **compatibility** in order to solve for the unknowns

Slope Deflection Method

Establish equilibrium equations “manually” along each DOF

Rotation DOFs:

Displacement DOFs:

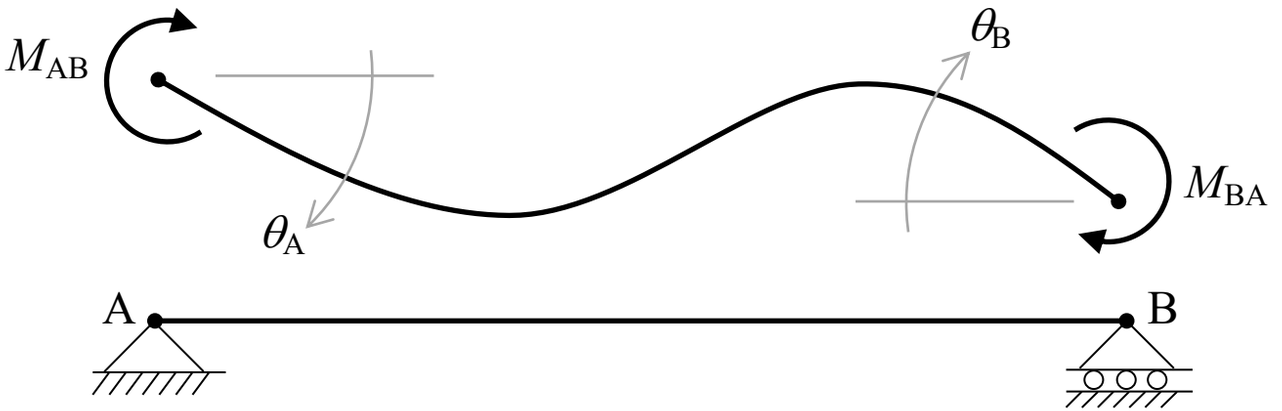


All equilibrium equations are formulated in terms of end moments

The unknowns are displacements & rotations

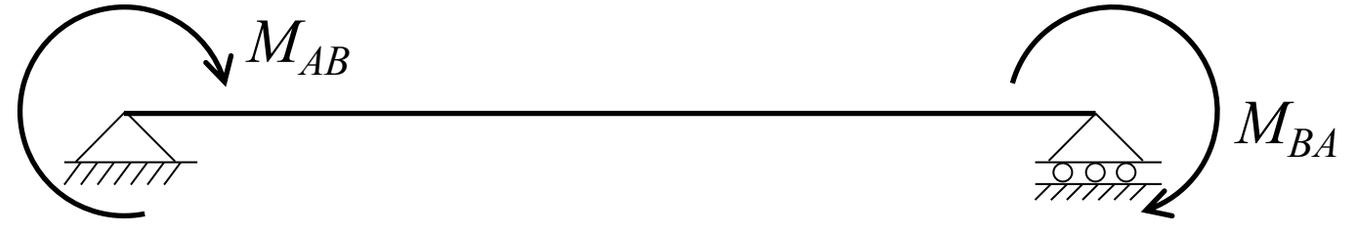
Slope Deflection Equation

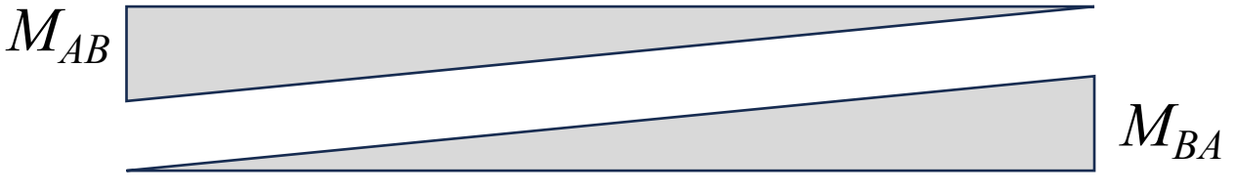
Relates end moments to displacements & rotations



Rotations

Derivation using virtual work

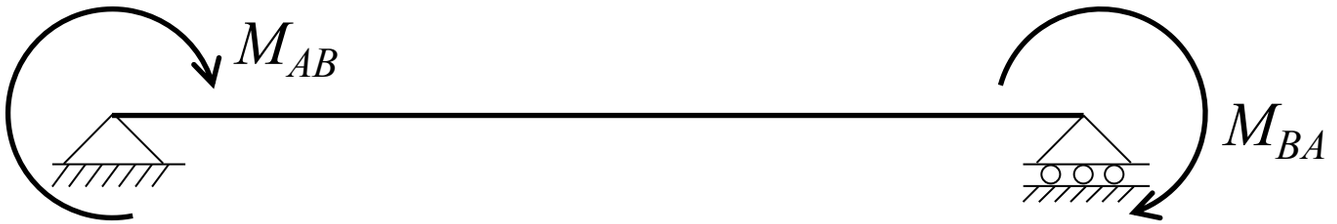




$$\theta_A = \frac{1}{3EI} \cdot 1 \cdot M_{AB} \cdot L - \frac{1}{6EI} \cdot 1 \cdot M_{BA} \cdot L$$

$$\theta_B = \frac{1}{3EI} \cdot 1 \cdot M_{BA} \cdot L - \frac{1}{6EI} \cdot 1 \cdot M_{AB} \cdot L$$

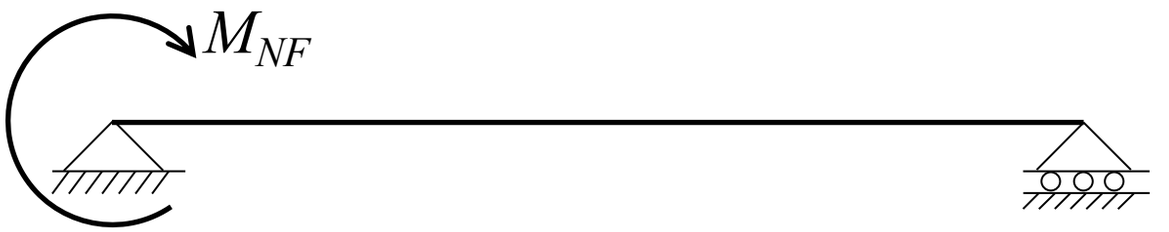
Solve for End Moments



$$M_{AB} = \frac{4EI}{L}\theta_A + \frac{2EI}{L}\theta_B = \frac{2EI}{L}\cdot\left(2\theta_A + \theta_B\right)$$

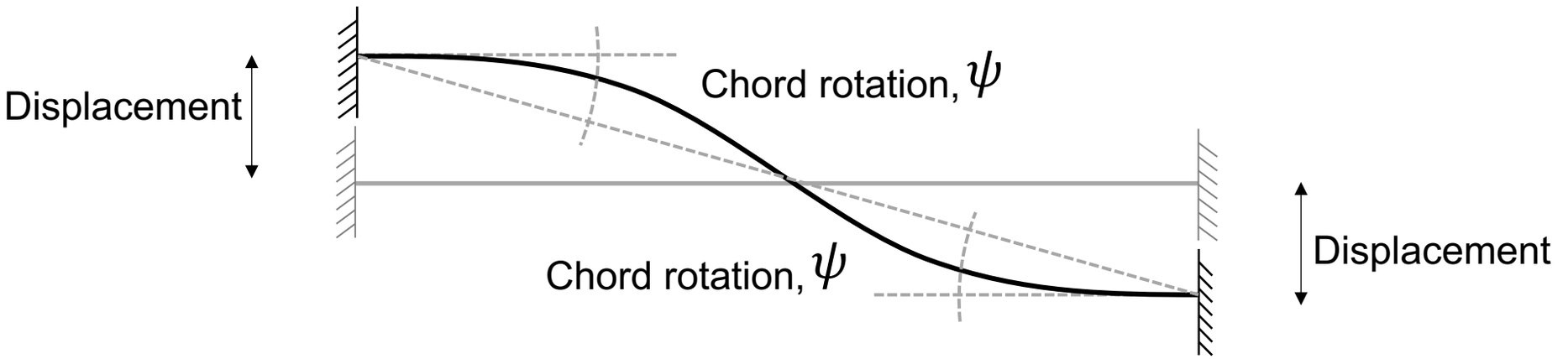
$$M_{BA} = \frac{4EI}{L}\theta_B + \frac{2EI}{L}\theta_A = \frac{2EI}{L}\cdot\left(2\theta_B + \theta_A\right)$$

Generalized Notation



$$M_{NF} = \frac{2EI}{L}(2\theta_N + \theta_F)$$

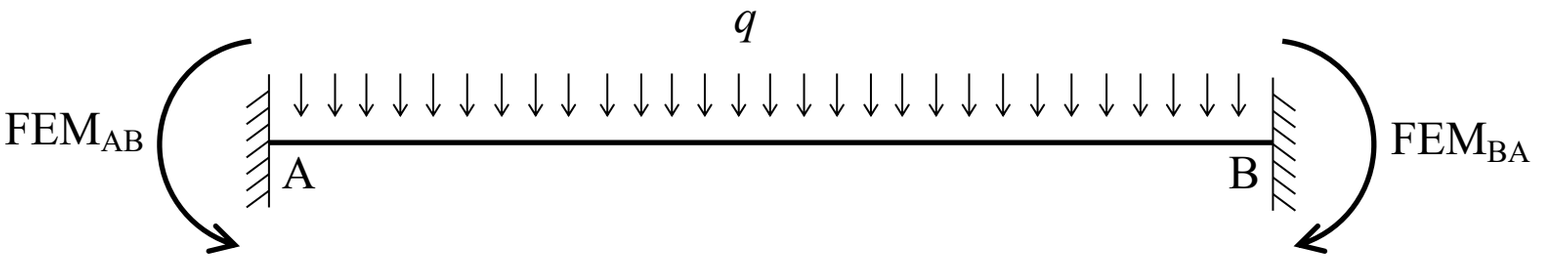
Displacements



$$M_{NF} = \frac{2EI}{L}(2\theta_N + \theta_F) = \frac{2EI}{L}\big(2(-\psi) + (-\psi)\big) = -\frac{2EI}{L} \cdot 3\psi$$

An additional source of end moments

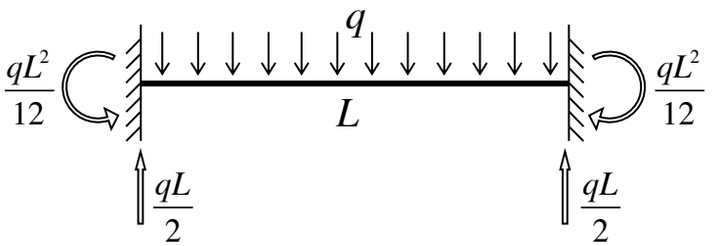
Fixed-end moments (FEM) from loading

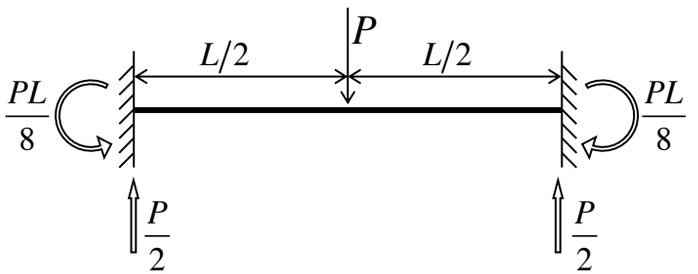


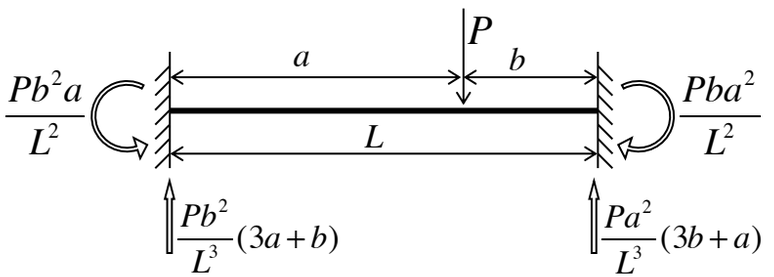
Final slope deflection equation:

$$M_{NF} = \frac{2EI}{L}(2\theta_N + \theta_F - 3\psi) + FEM_{NF}$$

Formula Sheet



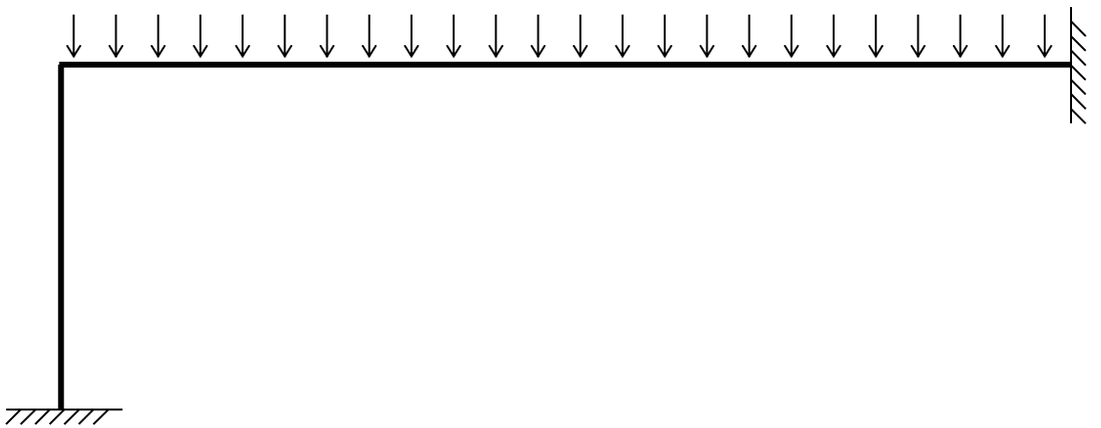




Procedure

1. Identify the degrees of freedom, i.e., the unknown displacements and rotations
2. Establish equilibrium equations in terms of **end moments**
3. Substitute slope-deflection equation for end moments
4. Solve for the unknown displacements and rotations
5. Substitute displacements and rotations into slope-deflection equation to get end moments
6. Draw bending moment diagram between known **end moments**

Example



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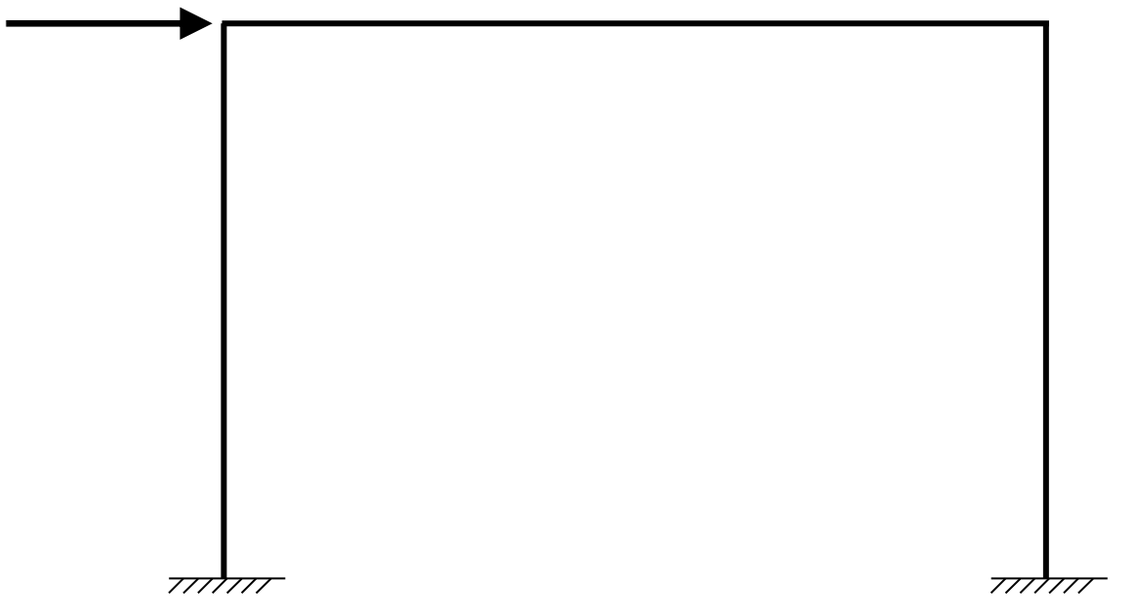

More lectures:

Terje's Toobox:

terje.civil.ubc.ca