A short course on

Probabilities and Random Variables

This video:

Total Probability Integration

Terje's Toolbox is freely available at terje.civil.ubc.ca
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Total Probability Rule

$$P(A) = \sum_{i=1}^{n} P(A \mid E_i) P(E_i)$$

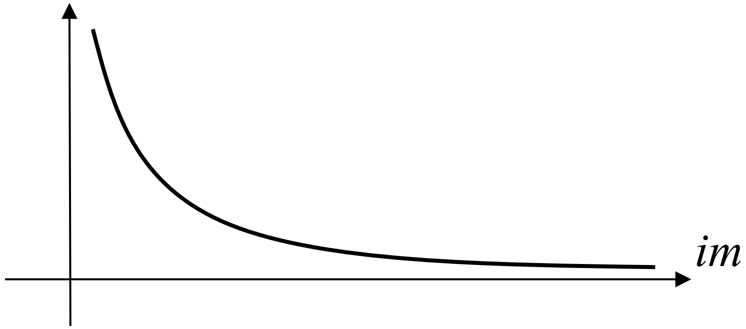
$$\mathrm{P}(A) = \int_{-\infty}^{\infty} \mathrm{P}(A \mid x) \cdot f(x) dx$$

$$f(x) = \int_{-\infty}^{\infty} f(x \mid y) \cdot f(y) dy$$

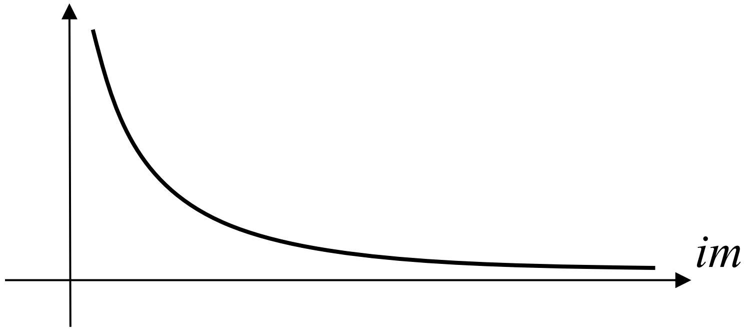
$$f(x) = \sum_{i=1}^{N} f(x \mid E_i) \cdot P(E_i)$$

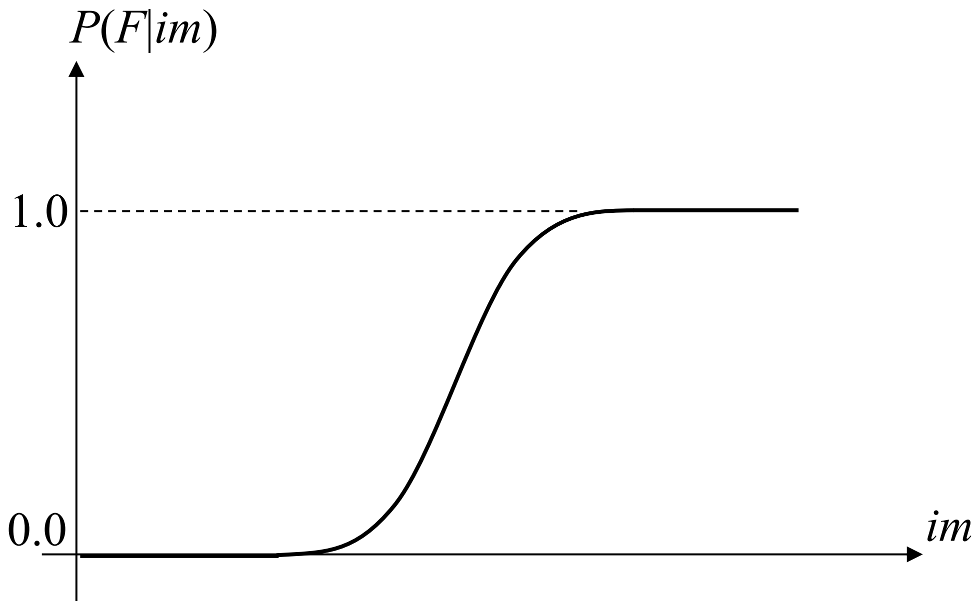
Hazard Curves & Fragility Functions

Probability of **exceedance**, $G(im)$



Annual rate of **exceedance**, $\lambda(im)$





Lognormal CDF as Fragility Function

$$F(x) = \Phi\left(\frac{1}{\sigma_Y}\ln\left(\frac{x}{m_X}\right)\right)$$

$$P(DS \geq ds_i) = \Phi\left(\frac{1}{\beta}\ln\left(\frac{d}{\theta}\right)\right)$$

$$m_{\ln(d)} = \mu_{\ln(d)} = \frac{1}{N}\cdot\sum_{i=1}^{N}\ln(d_i)$$

$$\theta = m_d = \exp\left(m_{\ln(d)}\right) = \exp\left(\frac{1}{N}\cdot\sum_{i=1}^{N}\ln(d_i)\right)$$

$$\beta^2 = \mathrm{Var}\left[\ln(d)\right] = \mathrm{Var}\left[\ln\left(\frac{d}{\theta}\right)\right] = \frac{1}{N-1}\cdot\sum_{i=1}^{N}\left(\ln\left(\frac{d_i}{\theta}\right)\right)^2$$

$$\beta = \sqrt{\frac{1}{N-1}\cdot\sum_{i=1}^{N}\left(\ln\left(\frac{d_i}{\theta}\right)\right)^2}$$

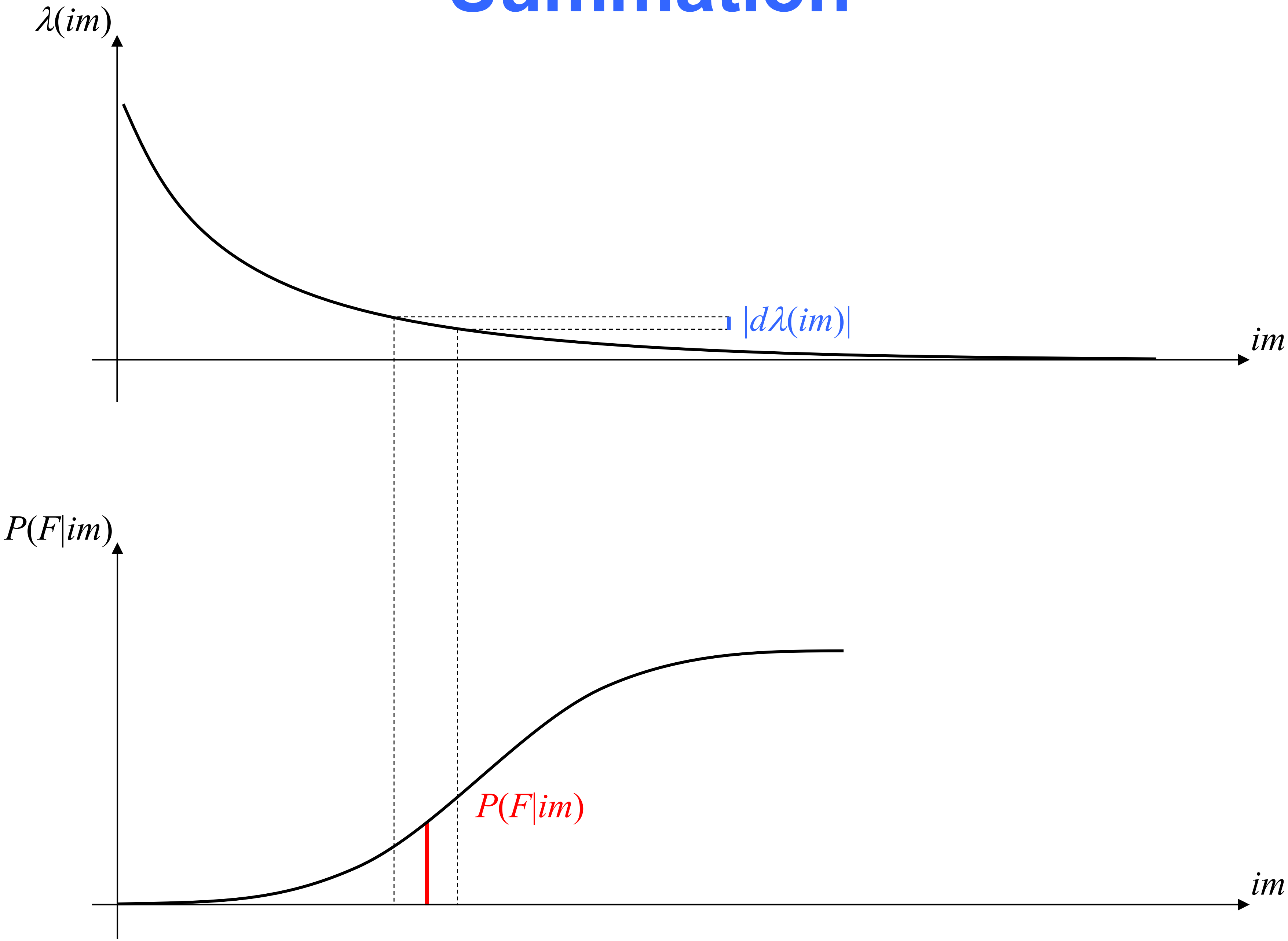
Total Probability Integration

Probability: $$P(F) = \int_0^\infty P(F \mid im) \cdot f(im) \cdot dim = \int_0^\infty P(F \mid im) \cdot \left| \frac{dG(im)}{dim} \right| \cdot dim = \int_0^\infty P(F \mid im) \cdot \left| dG(im) \right| = \sum_{im} P(F \mid im) \cdot \left| dG(im) \right|$$

Rate: $$\lambda(F) = \int_0^\infty P(F \mid im) \cdot \left| \frac{d\lambda(im)}{dim} \right| \cdot dim = \sum_{im} P(F \mid im) \cdot \left| d\lambda(im) \right|$$

Multiple sources: $$\lambda(F) = \sum_{s=1}^{K} \lambda(F \mid s) = \sum_{s=1}^{K} \left(\sum_{im} P(F \mid im, s) \cdot \left| d\lambda(im, s) \right| \right)$$

Summation



Via Damage

$$\lambda(d) = \int_0^\infty G(d \mid im) \cdot \left| \frac{d\lambda(im)}{dim} \right| \cdot dim = \int_0^\infty G(d \mid im) \cdot \left| d\lambda(im) \right|$$

$$\lambda(F) = \int_0^\infty \int_0^\infty P(F \mid d) \cdot \left| G(d \mid im) \right| \cdot \left| d\lambda(im) \right|$$

PEER Triple Integral

Notation: DV = decision variable, e.g., monetary loss
DM = damage measure
EDP = engineering demand parameter, e.g., interstorey drift
IM = intensity measure

Have: $G(dv|dm)$, $G(dm|edp)$, $G(edp|im)$, $\lambda(im)$

Want: $\lambda(dv)$

Total Probability Integration

$$\lambda(edp) = \int_0^\infty G(edp|im) \cdot \left|\frac{d\lambda(im)}{dim}\right| dim = \int_0^\infty G(edp|im) \cdot |d\lambda(im)|$$

$$\frac{\lambda(edp)}{dedp} = \int_0^\infty \frac{dG(edp|im)}{dedp} \cdot |d\lambda(im)|$$

$$\lambda(dm) = \int_0^\infty G(dm|edp) \cdot \left|\frac{d\lambda(edp)}{dedp}\right| dedp = \int_0^\infty G(dm|edp) \cdot |d\lambda(edp)|$$

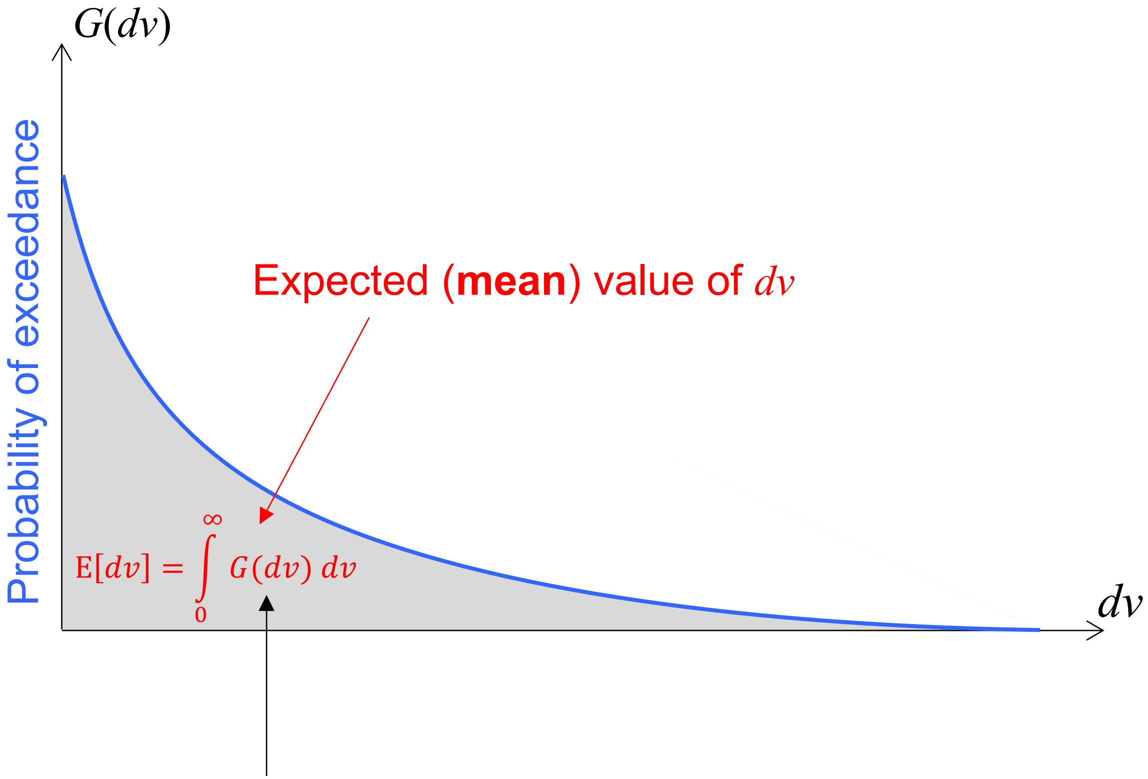
$$\frac{\lambda(dm)}{ddm} = \int_0^\infty \frac{dG(dm|edp)}{dedp} \cdot |d\lambda(edp)|$$

$$\lambda(dv) = \int_0^\infty G(dv|dm) \cdot \left|\frac{d\lambda(dm)}{ddm}\right| ddm = \int_0^\infty G(dv|dm) \cdot |d\lambda(dm)|$$

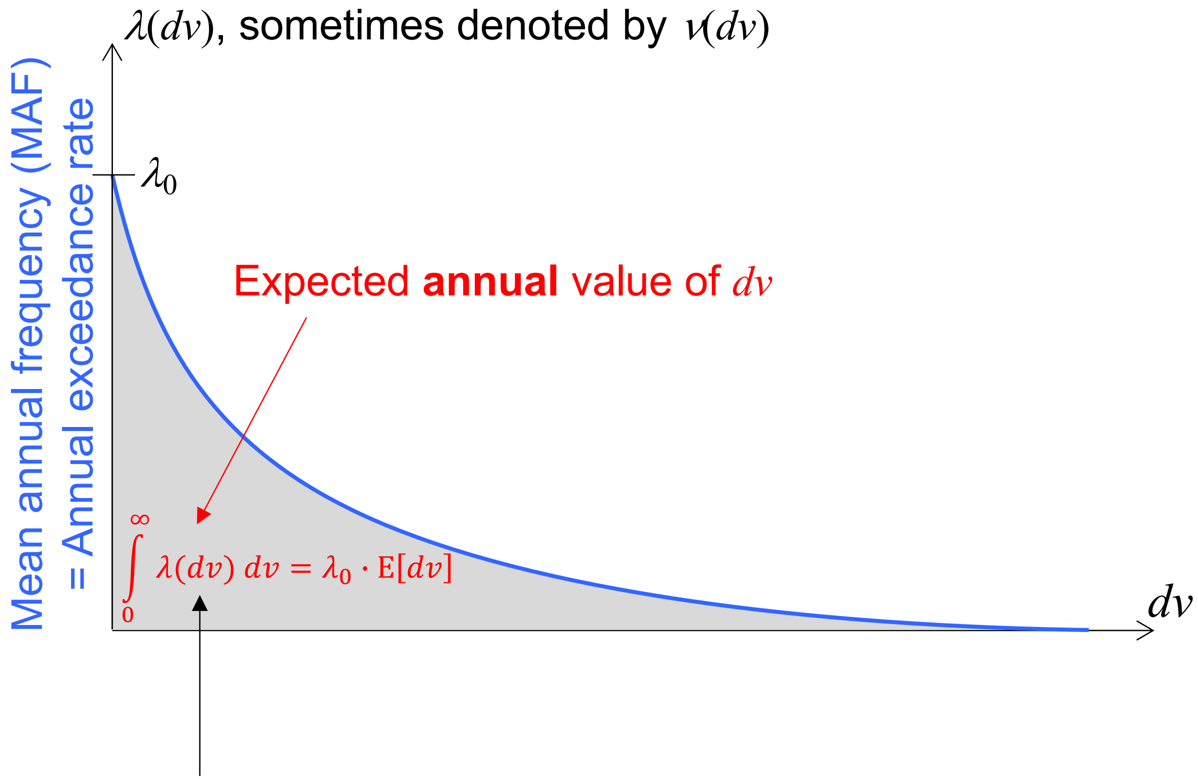
Conclusion: $$\lambda(dv) = \int_0^\infty \int_0^\infty \int_0^\infty G(dv|dm) \cdot |dG(dm|edp)| \cdot |dG(edp|im)| \cdot |d\lambda(im)|$$

Expectation

$$G = \frac{\lambda}{\lambda_0} \quad \leftrightarrow \quad \lambda = \lambda_0 \cdot G$$

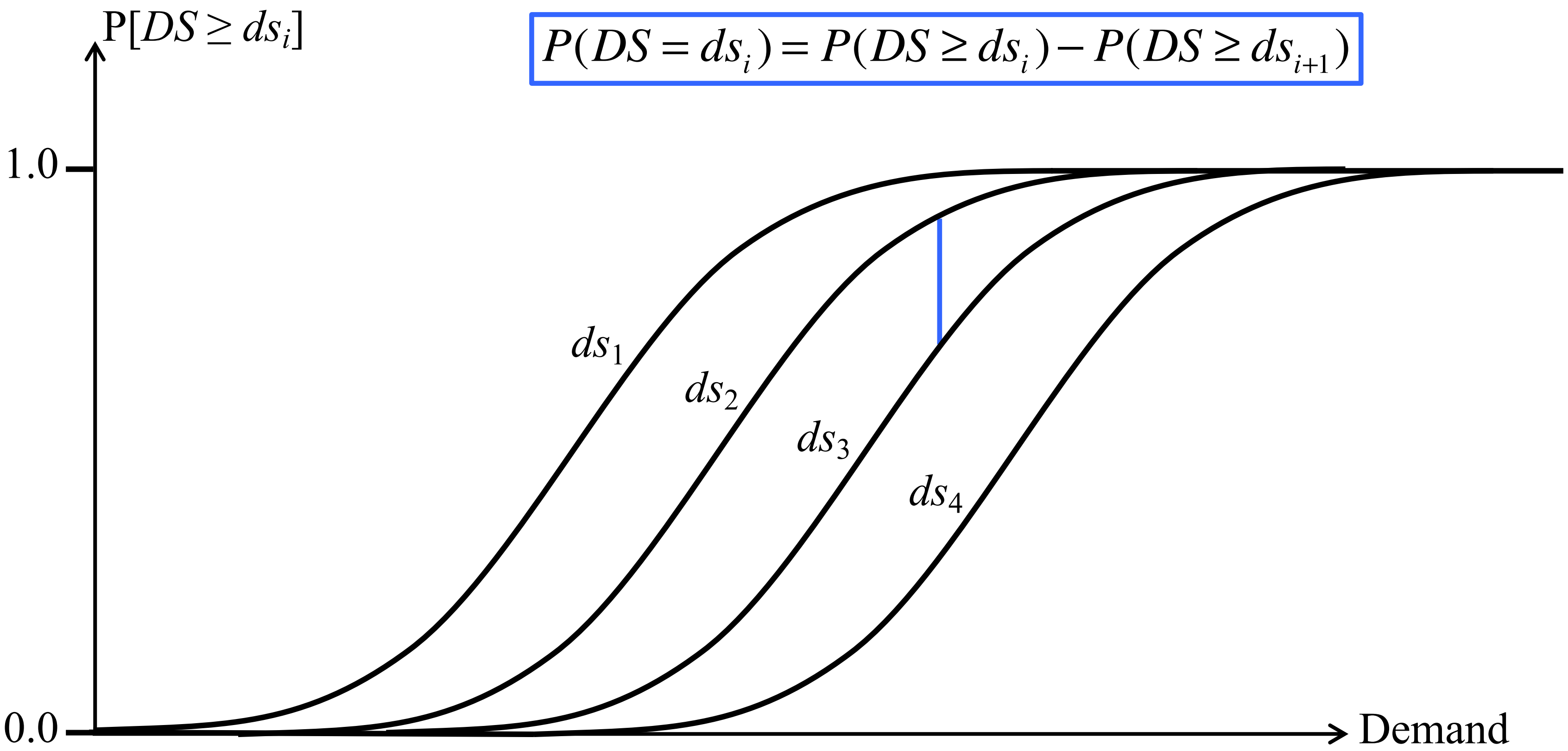


See random variables document



See Eq. (4) in ADKs non-ergodicity paper

Fragility Functions for Damage States



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Terje’s Toobox:

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