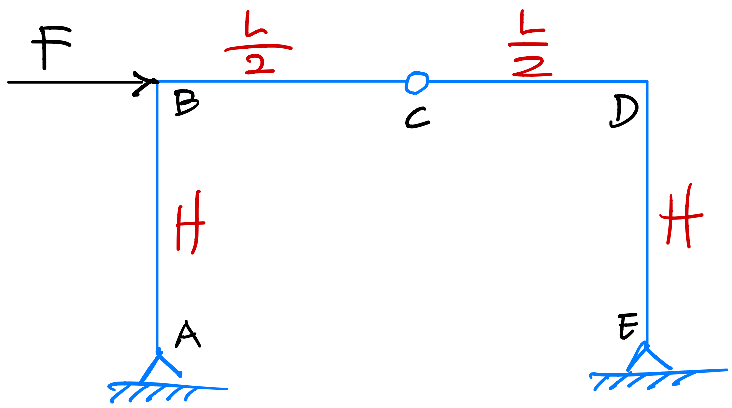
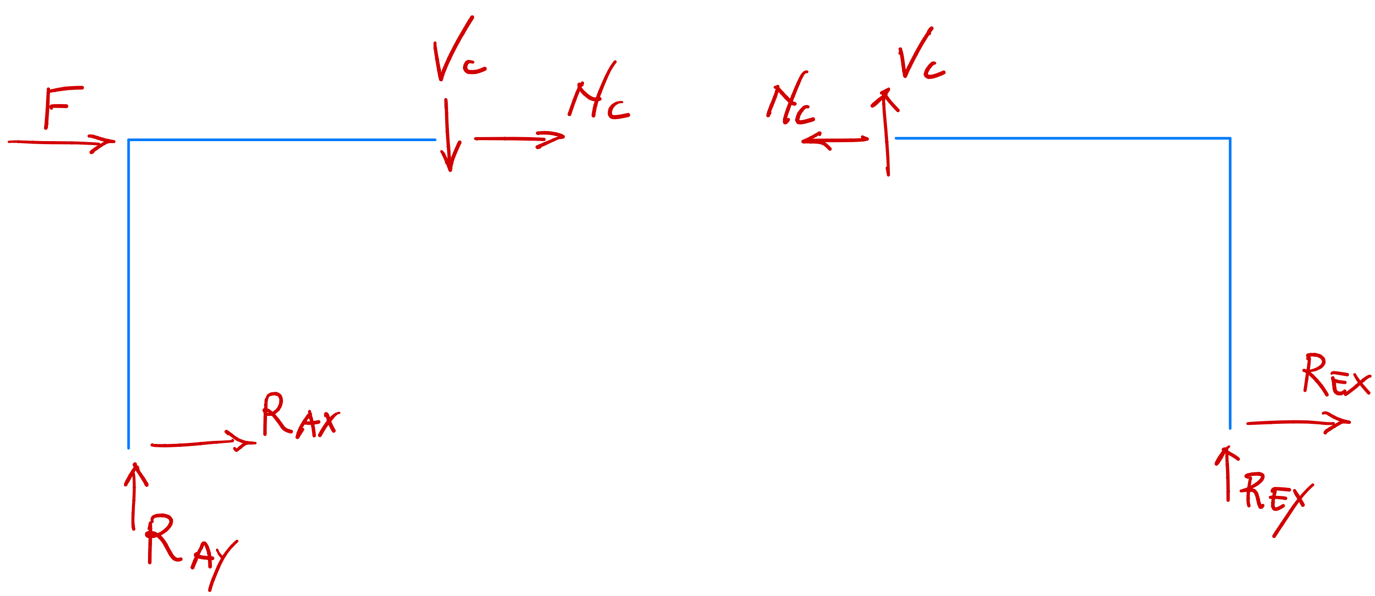
Example: Portal frame with hinge

Objective: Analyze a statically determinate structure with more than three support reactions.



$$DSI = (f \cdot m + s) - (e \cdot j + h) = (3 \cdot 4 + 4) - (3 \cdot 5 + 1) = \underline{0} \quad \text{(determinate)}$$

Free body diagrams:



The figure above shows a total of six unknown forces. We can set up three equilibrium equations for each part of the structure, for a total of six equations. They are shown on the next page, for reference:

Left part:

$$\sum F_x = F + R_{Ax} + N_c = 0$$

$$\sum F_y = R_{Ay} - V_c = 0$$

$$\sum M_A = F \cdot H + V_c \cdot \frac{L}{2} + N_c \cdot H = 0$$

Right part:

$$\sum F_x = R_{Ex} - N_c = 0$$

$$\sum F_y = R_{Ey} + V_c = 0$$

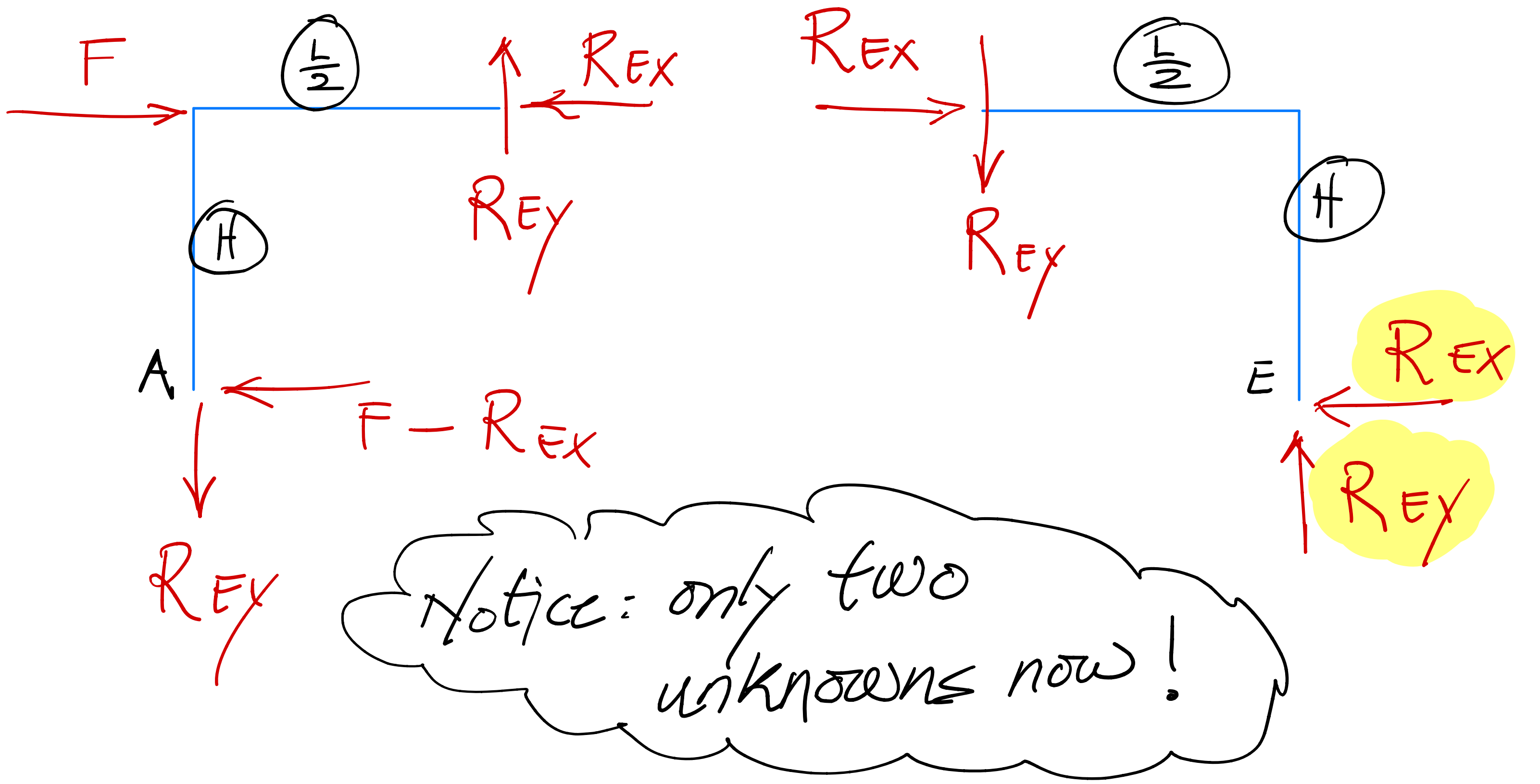
$$\sum M_E = V_c \cdot \frac{L}{2} - N_c \cdot H = 0$$

Although it does not add new information, we could also set up two global equations for the entire structure:

$$\sum F_x = F + R_{Ax} + R_{Ex} = 0$$

$$\sum F_y = R_{Ay} + R_{Ey} = 0$$

Solving the equations on the previous page simultaneously is a viable option. However, for hand calculations, we may want to observe horizontal and vertical equilibrium for the two parts first, without explicitly setting up the equations. Suppose we let the support reactions at E serve as the primary unknowns:



Two moment equilibrium equations:

$$① \quad \sum M_A = F \cdot H - R_{Ey} \cdot \frac{L}{2} - R_{Ex} H = 0$$

$$② \quad \sum M_E = R_{Ex} \cdot H - R_{Ey} \cdot \frac{L}{2} = 0$$

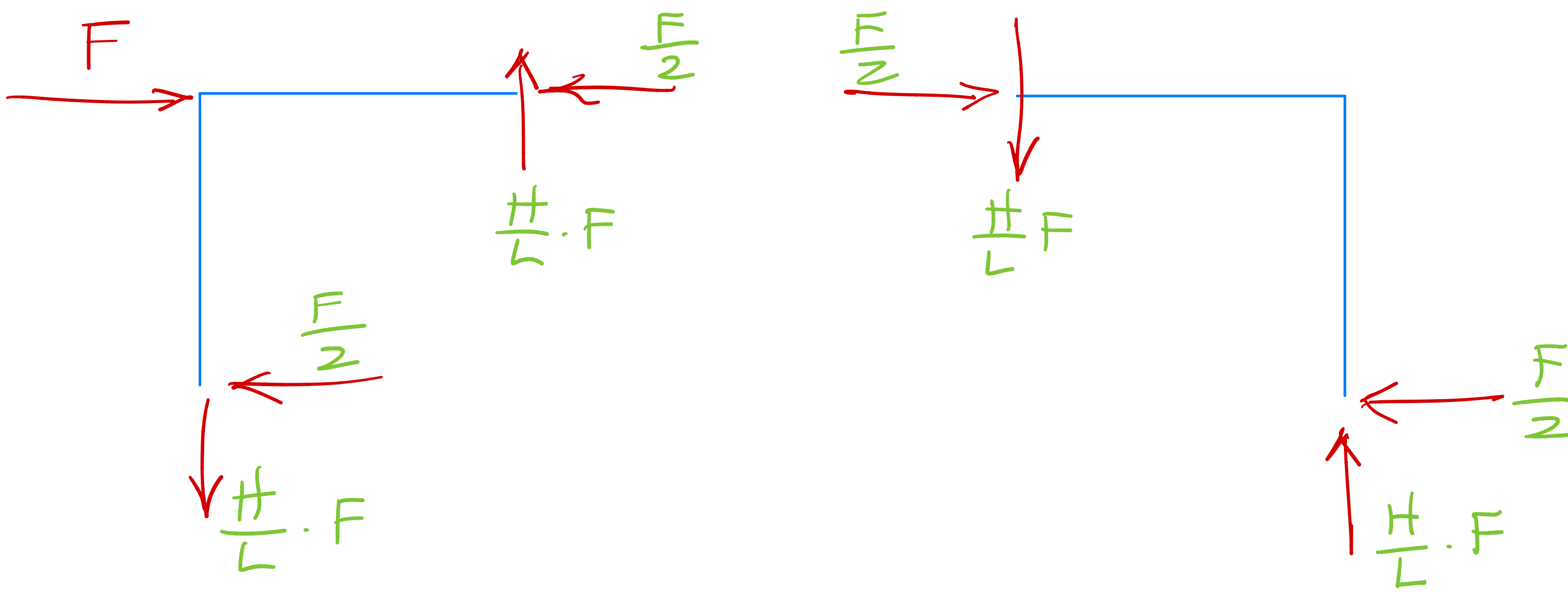
$$② \quad R_{Ex} = R_{Ey} \cdot \frac{L}{2H}$$

$$① \quad R_{Ey} = \frac{2HF}{L} - \frac{2H}{L} R_{Ex} = \frac{2HF}{L} - \frac{2H}{L} \cdot \frac{L}{2H} R_{Ey}$$

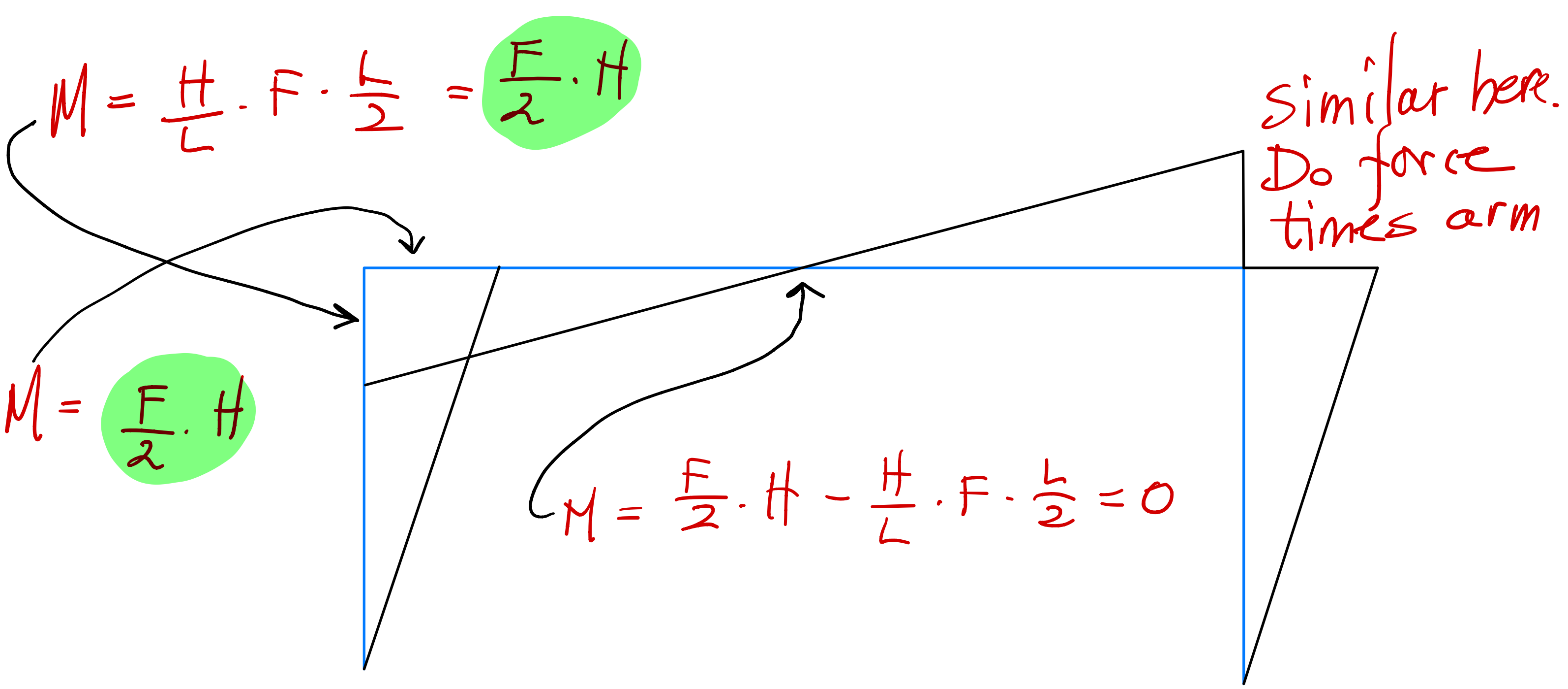
① $R_{Ey} = \frac{2HF}{L} - R_{Ey} \Longrightarrow R_{Ey} = \frac{H}{L} \cdot F$

② $R_{Ex} = \frac{HF}{L} \cdot \frac{L}{2H} = \frac{F}{2}$

Put those values onto the previous figure:



Resulting bending moment diagram drawn on the tension side:



(Simply draw straight lines between the calculated values.)