A short course on

Probabilities and Random Variables

This video:

Discrete Stochastic Processes

Terje's Toolbox is freely available at terje.civil.ubc.ca
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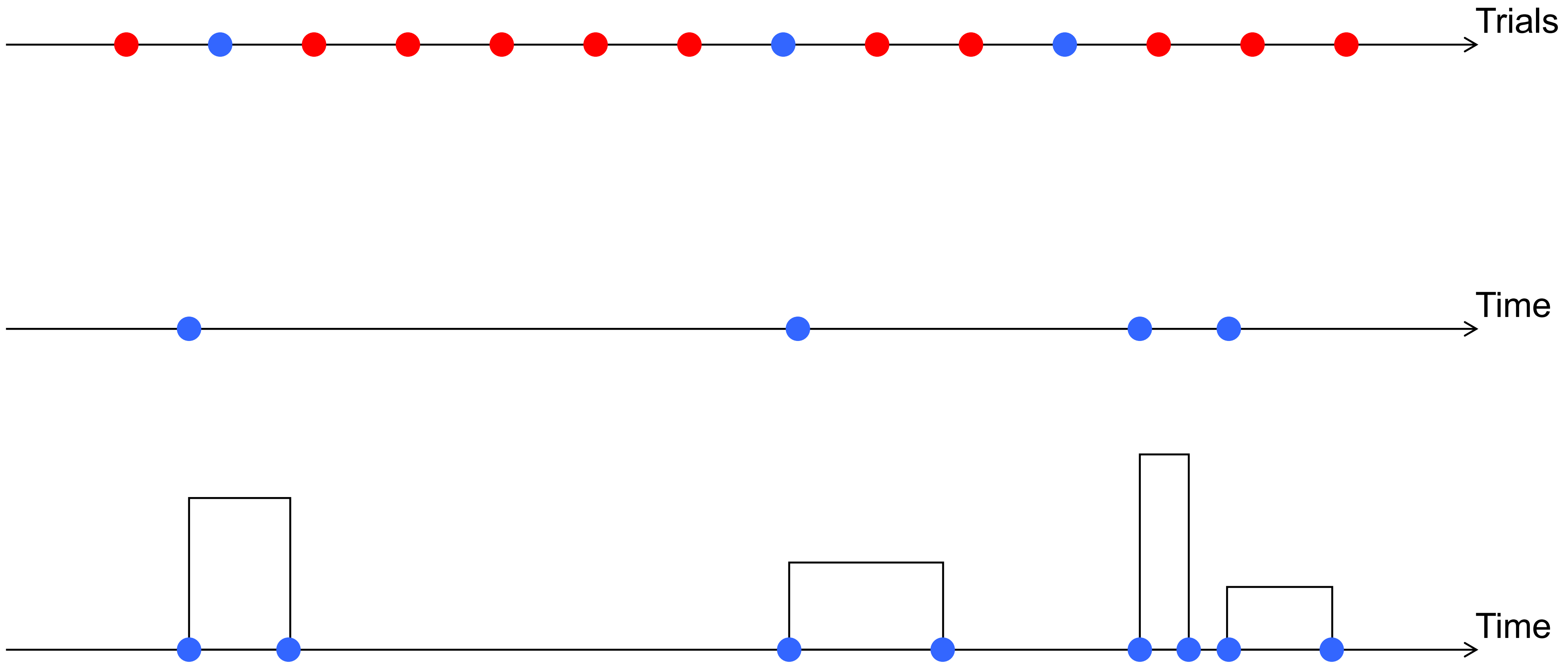
Bernoulli & Poisson



Bernoulli Trials

p = probability of "success" (the only model parameter)

n = number of trials

x = number of successes

s = number of trials between each success

Binomial PMF: $p(x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n \cdot (n-1) \cdot (n-2) \cdots (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdots (1)}$$

Geometric PMF: $p(s) = p \cdot (1-p)^{s-1}$

Poisson Point Process

Model: Let there be a Bernoulli trial at every infinitesimal time instant

Assumptions:
1) Constant rate of occurrence
2) Each occurrence is independent of past events
3) Only one occurrence at any given time

λ = rate of occurrence (the only model parameter)

T = time period under consideration

x = number of occurrences

t = time between occurrences

Poisson PMF: $p(x) = \frac{(\lambda \cdot T)^x}{x!} e^{-\lambda \cdot T}$

Exponential PDF: $f(t) = \lambda \cdot e^{-\lambda \cdot t}$

(We can work with rates without assuming the Poisson process, but then without linking it to probabilities)

Probability of “Occurrence”

$$\mathrm{P(occurrence)} = p(1) + p(2) + p(3) + \cdots$$

$$= 1 - p(0)$$

$$= 1 - e^{-\lambda \cdot T}$$

$$\Downarrow$$

$$\lambda = -\frac{\ln\big(1 - \mathrm{P(occurrence)}\big)}{T}$$

Return Period

Common expression in media after storms

Mean time between occurrences: $R \equiv \mu_t$ (large variability)

Inverse of the rate: $\mu_t = 1/\lambda$

Return period for 2% chance of occurrence in 50 years:

$$R = \frac{1}{\lambda} = -\frac{T}{\ln\left(1 - \mathrm{P(occurrence)}\right)} = -\frac{50}{\ln(1 - 0.02)} = 2{,}475 \text{ years}$$

Rate = Annual probability?

Return period, in years	Rate, i.e., mean annual frequency	Annual probability of occurrence
1	1	1/1.582
5	1/5	1/5.517
10	1/10	1/10.508
50	1/50	1/50.502
100	1/100	1/100.501
500	1/500	1/500.500
1,000	1/1,000	1/1000.500
10,000	1/10,000	1/10,000.500

Three ways to specify a Poisson process

Rate

Return period

Probability of occurrence in a time period

Derived Processes

Given two sources of earthquakes with λ_1 and λ_2

Rate of occurrence of any earthquake: $\lambda_1+\lambda_2$

Given failure probability, p_f, associated with a hazard with rate λ

Rate of failure: $\lambda \cdot p_f$

More lectures:

Terje’s Toobox:

terje.civil.ubc.ca