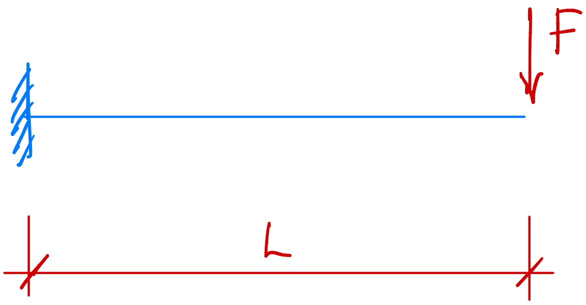
Example: Cantilevered beam

Objective: Getting into the habit of drawing the bending moment diagram (BMD) without setting up equations, but rather directly using equilibrium. In other words, use "moment = force * arm"

The simplest case possible is a point load:



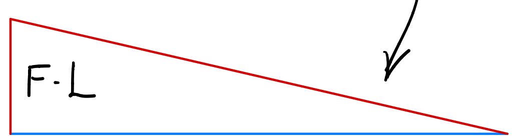
$M = F \cdot L$

$M = 0$ because free end

This "structure" is so simple that it isn't even necessary to find the reactions first

Draw BMD between known values

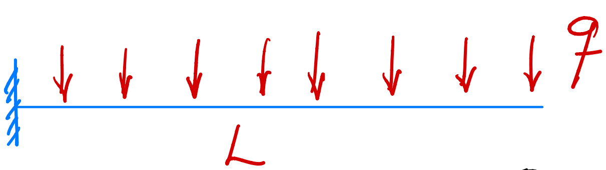
BMD:



Drawn on tension side

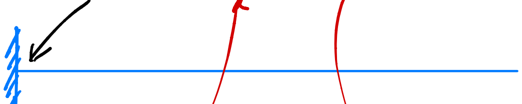
Could you easily tell which side of the beam is in tension?

Now consider uniformly distributed load (UDL):



$M = (q \cdot L) \cdot \frac{L}{2} = \frac{qL^2}{2}$

$M = 0$



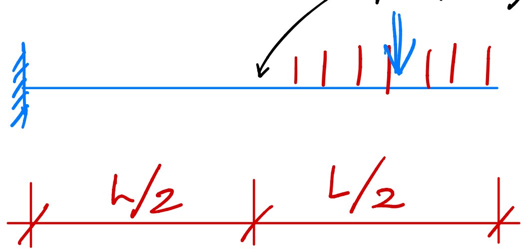
resultant

"arm"

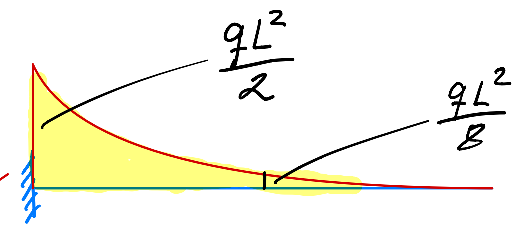
To make a couple of points related to the shape of the BMD, calculate the bending moment at the middle of the beam:

Do you understand this?

$$M = \underbrace{\left(q\frac{L}{2}\right)}\left(\frac{L}{4}\right) = \frac{qL^2}{8}$$

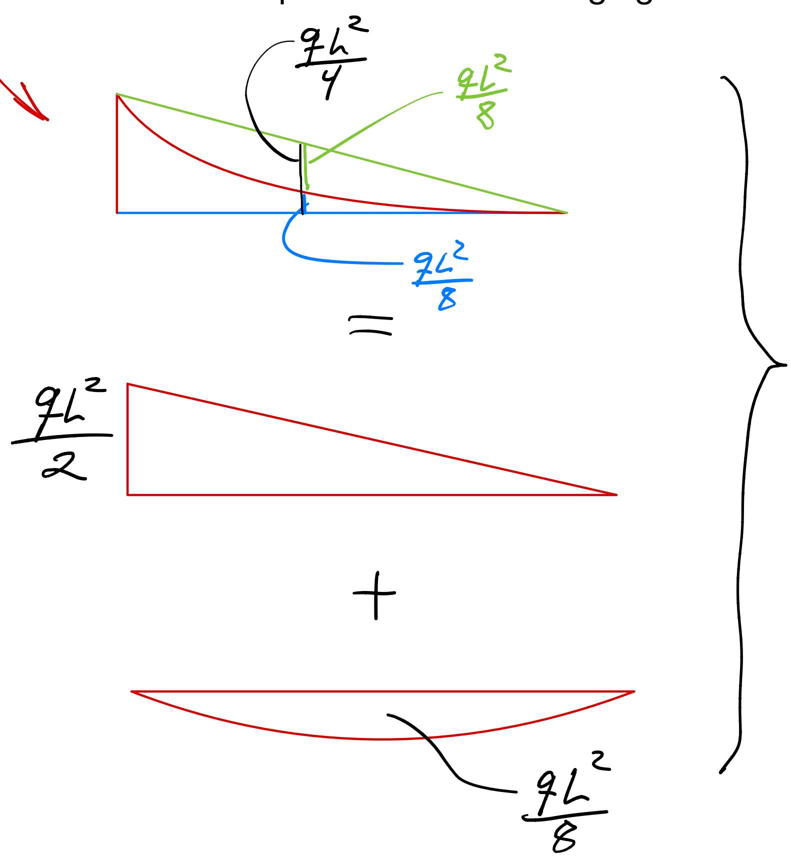


That leads to the following BMD:

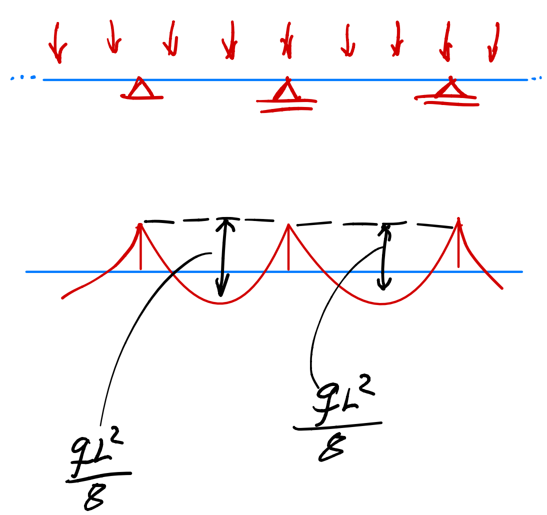


This value isn't really necessary, but it shows how the bending moment value is diminishing away from the support.

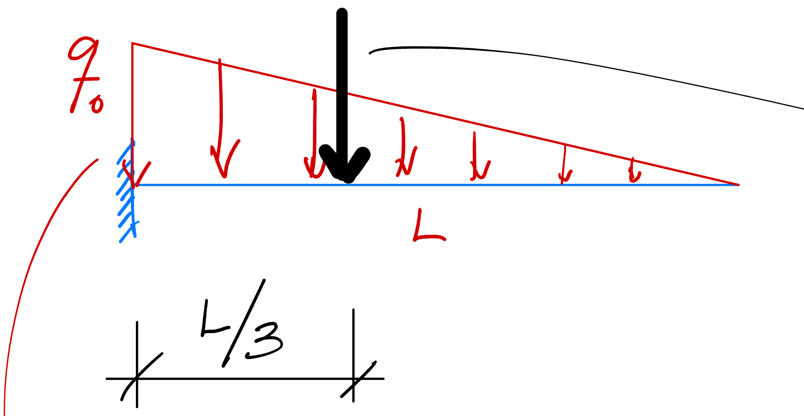
Having the moment value at the middle of the beam also allows the following point to be made: The convex parabola that is hanging between the end points has amplitude qL^2/8:



This knowledge is helpful when we do the Force Method later in the course. Also, notice how all structures with uniformly distributed load have these ql^2/8 parabolas:

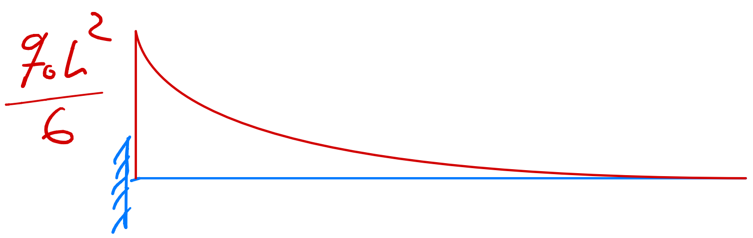


Although we mostly use uniformly distributed loads (or point loads) in the examples in this course, it is definitely possible to handle triangular load patterns in the same way:



Resultant : $\frac{1}{2} q_0 \cdot L$

$$M = \left(\frac{1}{2} q_0 L\right)\left(\frac{L}{3}\right) = \frac{q_0 L^2}{6}$$



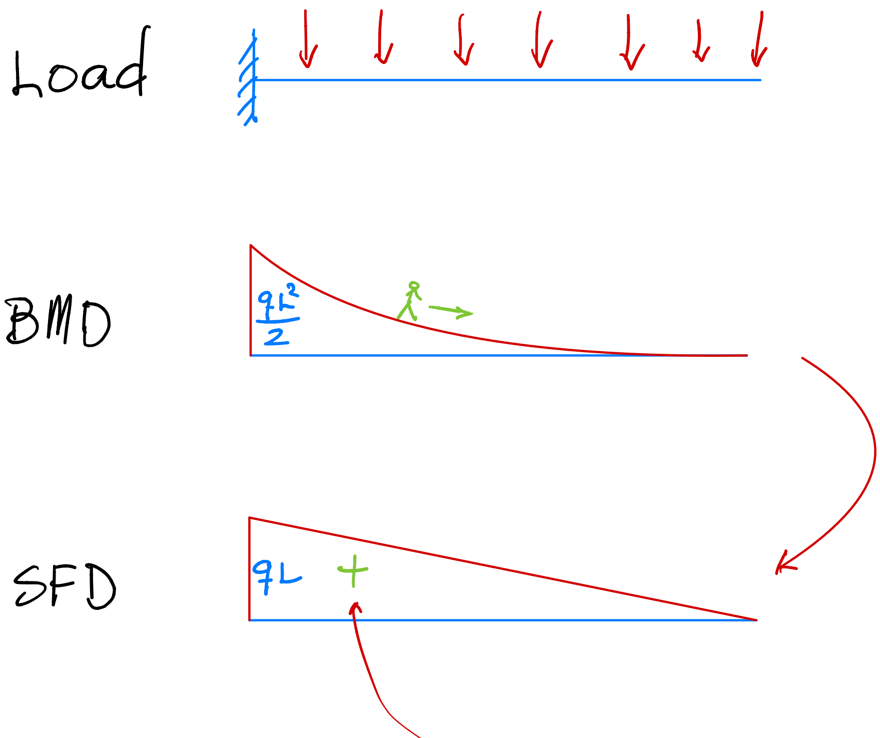
BMD drawn on the tension side

The shear force diagram (SFD) is obtained by differentiating the bending moment diagram: V=dM/dx. First consider the case of a point load:

$$V = \frac{\Delta M}{L} = \frac{FL}{L} = F$$

How to know whether the shear force is positive (clockwise) or negative (counter-clockwise)? Answer: Because we drawn the bending moment diagram on the TENSION SIDE a “downwards walk” means positive shear. That holds true regardless of whether the member is vertical or horizontal, or at an incline, as long as we walk from left to right, i.e., the way we walk along any mathematical function f(x).

Shear force diagram for the case of uniformly distributed load:



A couple of ways to think of this calculation: Formulate a function for M(x) and differentiate it to obtain V(x). Or, just look at the reaction value (qL) on the left-hand side. On the right-hand side the slope of the BMD is clearly zero, so the shear is zero.

Positive shear because of the “downhill” walk along the BMD.