A short course on

# Indeterminate Structures

This video:

**Moment Area Method**

Terje's Toolbox is freely available at terje.civil.ubc.ca
It is created and maintained by Professor Terje Haukaas, Ph.D., P.Eng.,
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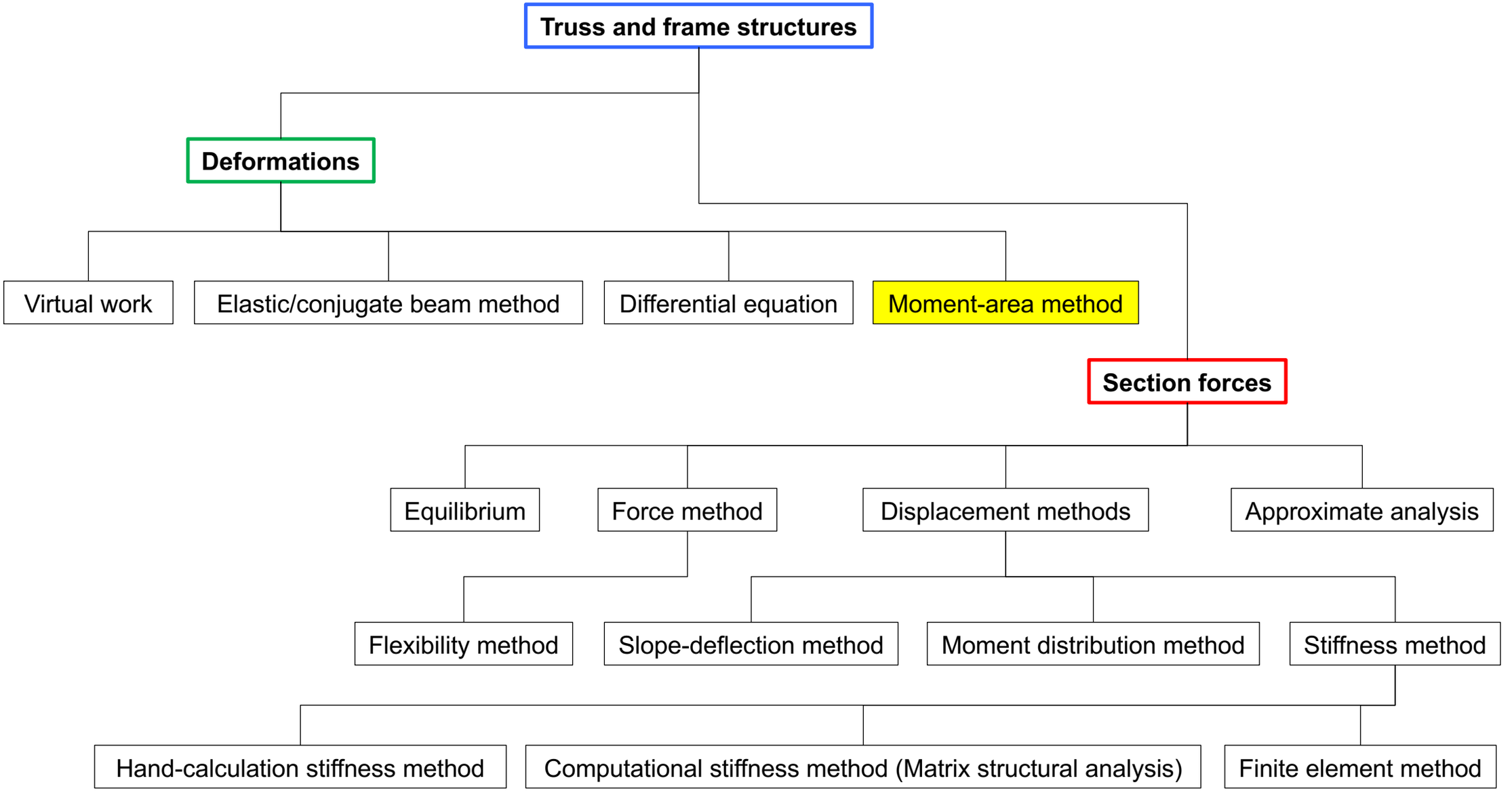
# Overview of Methods



# Application

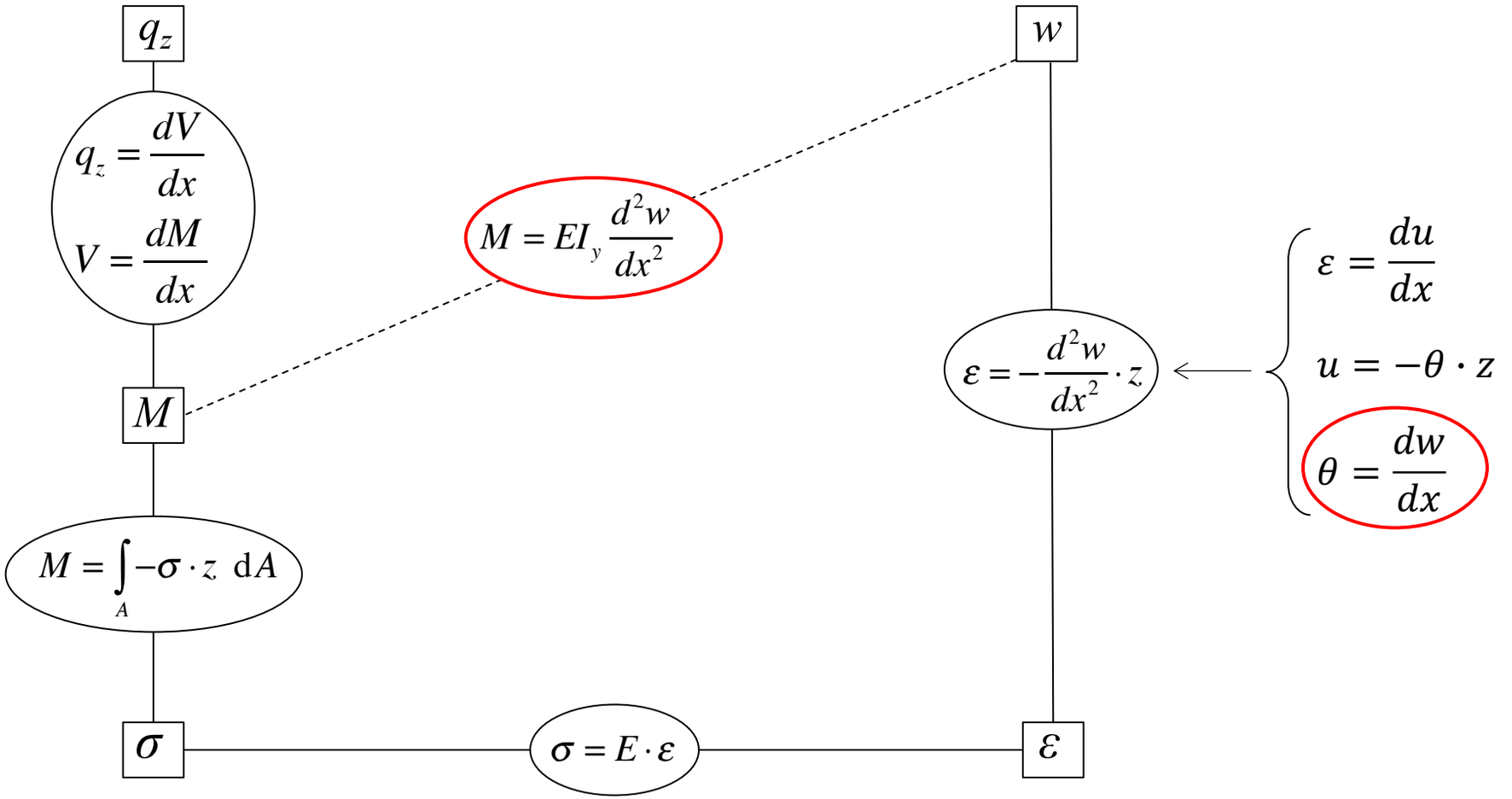
Known BMD

No need to re-analyze the structure

(Virtual work: Need to find BMD due to unit load at sought deformation)

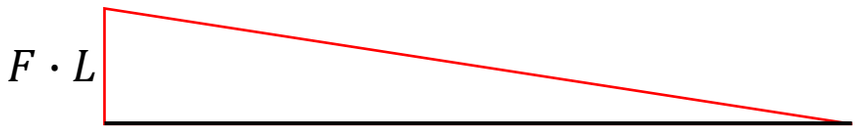
Ideal after finding the BMD for indeterminate structures

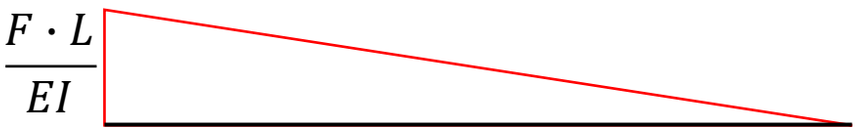
# Beam Equations



# Theorem 1

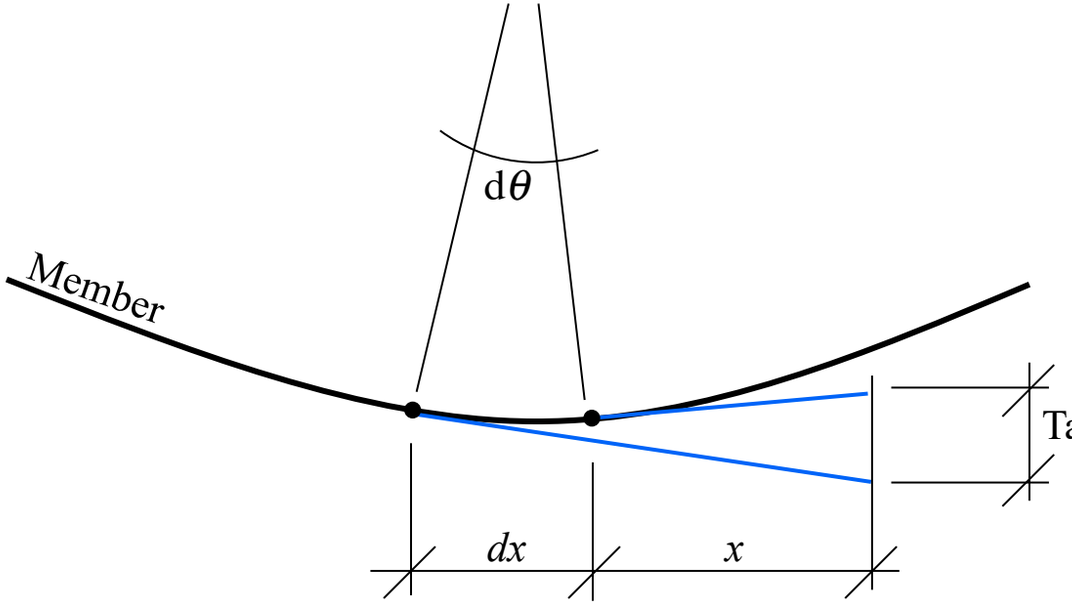

$$\frac{d\theta}{dx} = \frac{M}{EI} \quad \dashrightarrow \quad d\theta = \frac{M}{EI} \cdot dx$$




$$\theta_{AB} = \int_A^B d\theta = \int_A^B \frac{M}{EI} \cdot dx = \frac{1}{2} \cdot \left(\frac{F \cdot L}{EI}\right) \cdot L = \frac{FL^2}{2EI}$$

# Theorem 2

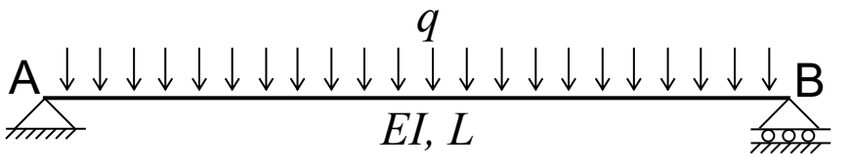


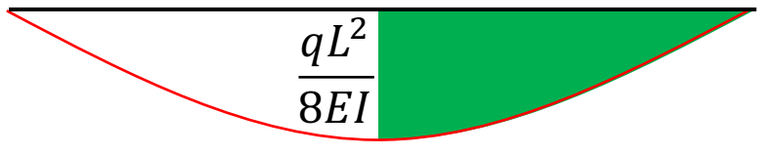

$$d\theta = \frac{M}{EI} \cdot dx$$

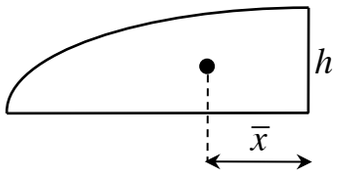
Tangential deviation, $dt \approx d\theta \cdot x = \frac{M}{EI} \cdot x \cdot dx$

$$t_{BA} = \int_A^B dt = \int_A^B \frac{M}{EI} x \quad dx$$

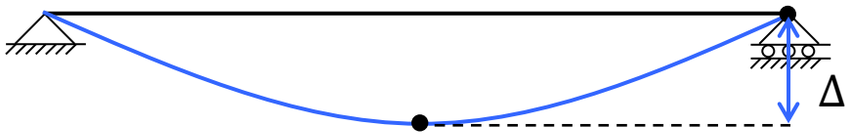
# Example



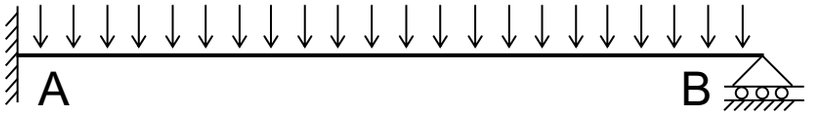

| Shape with length $L$ | Area | Centroid location |
|---|---|---|
|  | $A = \frac{2 \cdot h \cdot L}{3}$ | $\bar{x} = \frac{3L}{8}$ |

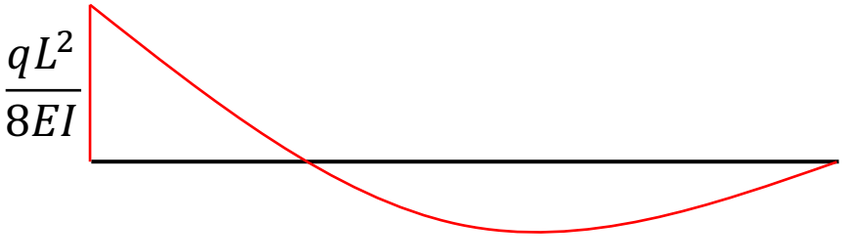


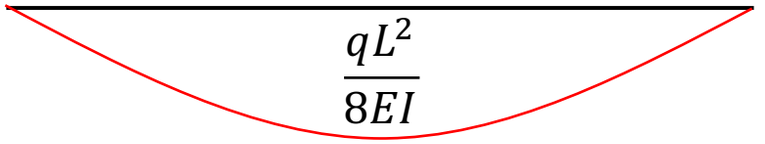


$$\Delta = t_{B\ midspan} = \int_{midspan}^{B} \frac{M}{EI} \cdot x\, dx = \left[\frac{2}{3} \cdot \left(\frac{qL^2}{8EI}\right) \cdot \frac{L}{2}\right] \cdot \left(\frac{5 \cdot \frac{L}{2}}{8}\right) = \frac{5qL^4}{384EI}$$

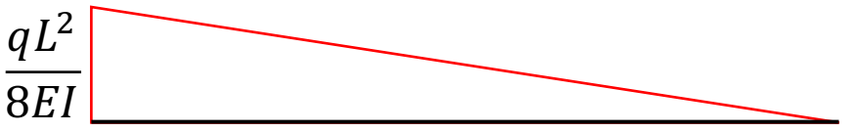
# Example



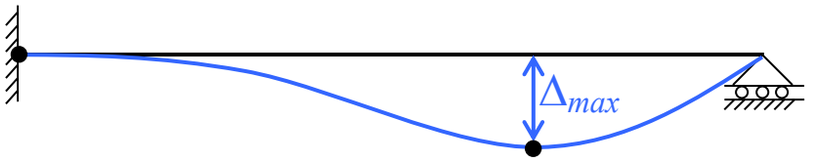




$$M_{par} = \frac{qL}{2EI} \cdot x - \frac{q}{2EI} \cdot x^2$$




$$M_{tri} = \frac{qL}{8EI} \cdot x - \frac{qL^2}{8EI}$$




$$\theta_B = \frac{2}{3} \cdot \frac{qL^2}{8EI} \cdot L - \frac{1}{2} \cdot \frac{qL^2}{8EI} \cdot L = \frac{qL^3}{48EI}$$

$$\theta_{A\,x_0} = \int_0^{x_0} \left(M_{par} + M_{tri}\right) dx = 0 \rightarrow x_0 = \frac{L}{16} \cdot \left(15 - \sqrt{33}\right)$$

$$\Delta_{max} = \int_0^{x_0} \left(M_{par} + M_{tri}\right) \cdot (x_0 - x) dx = \frac{\left(39 + 55 \cdot \sqrt{33}\right)}{65{,}536} \cdot \frac{qL^4}{EI} \approx \frac{qL^4}{184.6 \cdot EI}$$

More lectures:

Terje's Toobox:

terje.civil.ubc.ca