A short course on

Cross-section Analysis

This video:

Omega Diagram for the Calculation of Cross-section Constant and Stress in Warping Torsion

Terje’s Toolbox is freely available at terje.civil.ubc.ca
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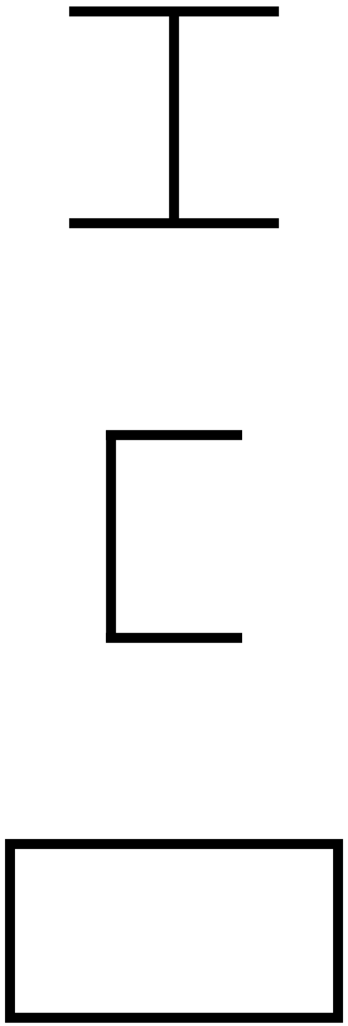
Scope

y_o, z_o	=	centroid coordinates
y_{sc}, z_{sc}	=	shear centre coordinates
A	=	cross-section area
I_y, I_z	=	moments of inertia
I_{yz}	=	product of inertia
θ	=	orientation of principal axes
J	=	Saint Venant torsion constant
Ω	=	omega diagram
C_w	=	warping torsion constant
A_{vy}, A_{vz}	=	shear area
σ	=	axial stress
τ	=	shear stress
q_s	=	shear flow

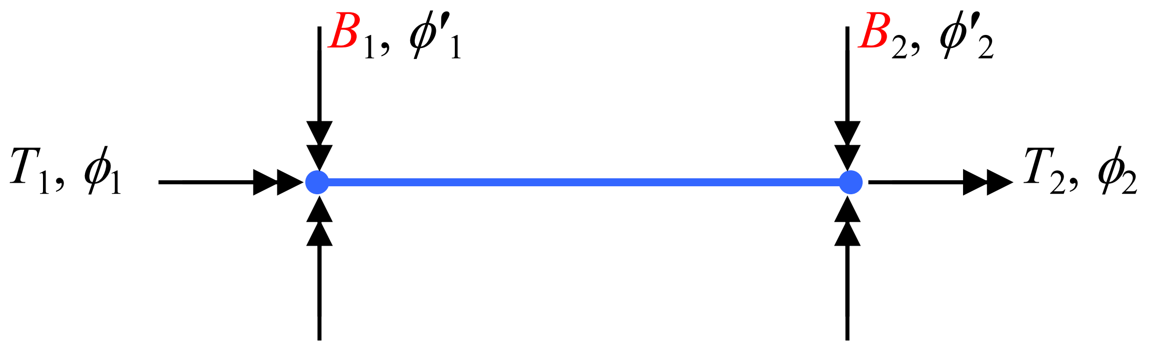
Warping Torsion



$$C_w = \int_A \Omega^2 \, dA$$

$$\sigma = \frac{B}{C_w} \cdot \Omega$$

$$B = -E C_w \cdot \phi'$$

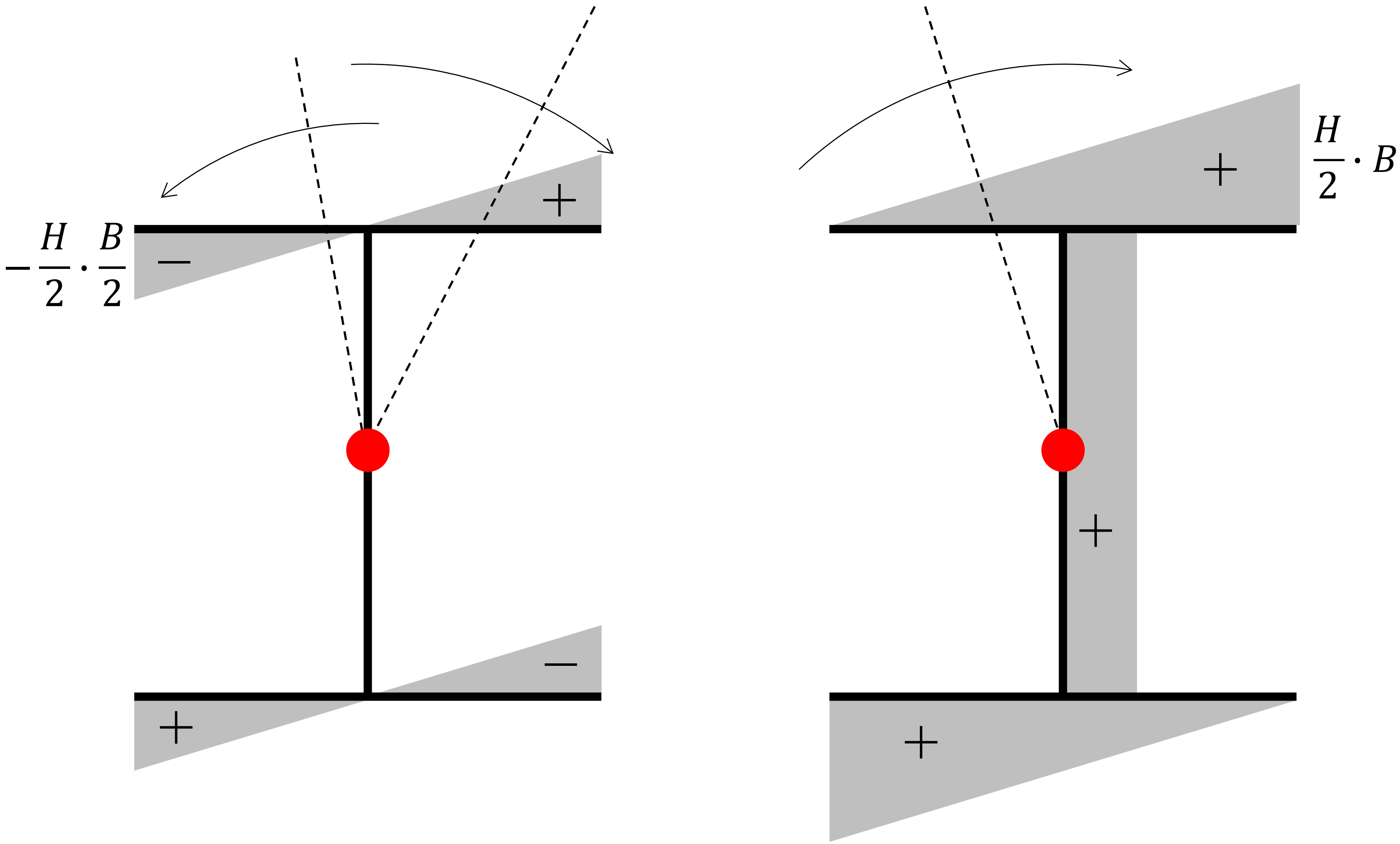


Omega Diagrams

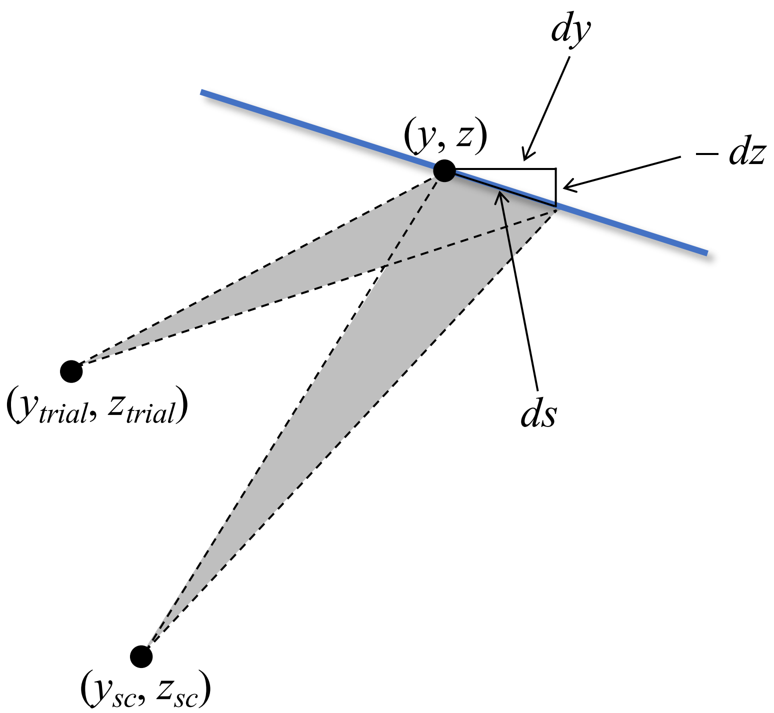
$$\Omega \equiv \int h\,ds$$

$$\int_A \Omega \;\mathrm{d}A = 0$$

$$\int_A y \cdot \Omega \;\mathrm{d}A = \int_A z \cdot \Omega \;\mathrm{d}A = 0$$



Omega Diagram



$$d\Omega_{sc} = dy\cdot\left(z - z_{sc}\right) - dz\cdot\left(y - y_{sc}\right)$$

$$d\Omega_{trial} = dy\cdot\left(z - z_{trial}\right) - dz\cdot\left(y - y_{trial}\right)$$

$$d\Omega_{sc} - d\Omega_{trial} = \left(y_{sc} - y_{trial}\right)\cdot dz - \left(z_{sc} - z_{trial}\right)\cdot dy$$

$$\Omega_{sc} - \Omega_{trial} = \left(y_{sc} - y_{trial}\right)\cdot z - \left(z_{sc} - z_{trial}\right)\cdot y + C$$

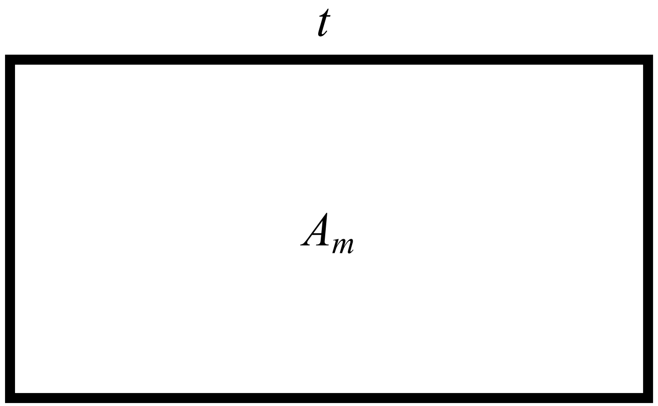
$$\boxed{\Omega = \Omega_{trial} + \left(y_{sc} - y_{trial}\right)\cdot z - \left(z_{sc} - z_{trial}\right)\cdot y + C}$$

$$\int_A \Omega\, dA = \int_A \Omega_{trial}\, dA + \int_A C\, dA = 0 \quad\Rightarrow\quad \boxed{C = -\frac{\int_A \Omega_{trial}\, dA}{A}}$$

$$\int_A y\cdot\Omega\, dA = \int_A y\cdot\Omega_{trial}\, dA - \left(z_{sc} - z_{trial}\right)\cdot\int_A y^2\, dA = 0 \quad\Rightarrow\quad \boxed{z_{sc} - z_{trial} = \frac{\int_A y\cdot\Omega_{trial}\, dA}{I_z}}$$

$$\int_A z\cdot\Omega\, dA = \int_A z\cdot\Omega_{trial}\, dA + \left(y_{sc} - y_{trial}\right)\cdot\int_A z^2\, dA = 0 \quad\Rightarrow\quad \boxed{y_{sc} - y_{trial} = -\frac{\int_A z\cdot\Omega_{trial}\, dA}{I_y}}$$

Closed Cross-sections

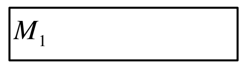
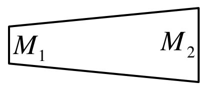
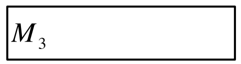
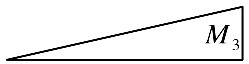
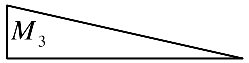
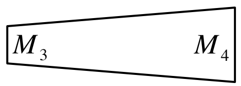
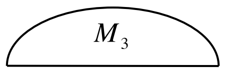
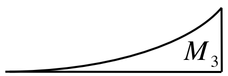
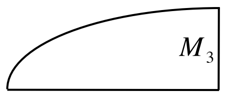


$$\Omega \equiv \int \left(h - \bar{h} \right) ds$$

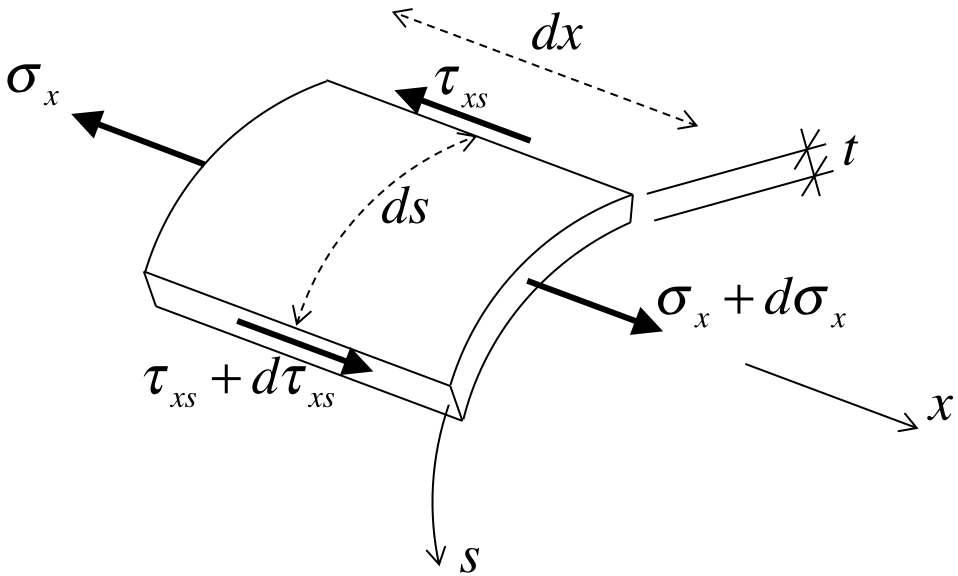
$$\bar{h} = \frac{J}{2 \cdot t \cdot A_m}$$

$$C_w = \int_A \Omega^2 \, dA$$

$$\sigma = \frac{B}{C_w} \cdot \Omega$$

$\Delta = \int_0^L \delta M \cdot \frac{M}{EI} dx$				
	$\frac{1}{EI}M_1M_3L$	$\frac{1}{2EI}M_1M_3L$	$\frac{1}{2EI}(M_1+M_2)M_3L$	$\frac{1}{2EI}M_1M_3L$
	$\frac{1}{2EI}M_1M_3L$	$\frac{1}{3EI}M_1M_3L$	$\frac{1}{6EI}(M_1+2M_2)M_3L$	$\frac{1}{4EI}M_1M_3L$
	$\frac{1}{2EI}M_1M_3L$	$\frac{1}{6EI}M_1M_3L$	$\frac{1}{6EI}(2M_1+M_2)M_3L$	$\frac{1}{4EI}M_1M_3L$
	$\frac{1}{2EI}M_1(M_3+M_4)L$	$\frac{1}{6EI}M_1(M_3+2M_4)L$	$\frac{1}{6EI}M_1(2M_3+M_4)L$ $+\frac{1}{6EI}M_2(M_3+2M_4)L$	$\frac{1}{4EI}(M_1M_3+M_1M_4)L$
	$\frac{1}{2EI}M_1M_3L$	$\frac{1}{4EI}M_1M_3L$	$\frac{1}{4EI}(M_1M_3+M_2M_3)L$	$\frac{1}{3EI}M_1M_3L$
	$\frac{2}{3EI}M_1M_3L$	$\frac{1}{3EI}M_1M_3L$	$\frac{1}{3EI}(M_1+M_2)M_3L$	$\frac{5}{12EI}M_1M_3L$
	$\frac{1}{3EI}M_1M_3L$	$\frac{1}{4EI}M_1M_3L$	$\frac{1}{12EI}(M_1+3M_2)M_3L$	$\frac{7}{48EI}M_1M_3L$
	$\frac{2}{3EI}M_1M_3L$	$\frac{5}{12EI}M_1M_3L$	$\frac{1}{12EI}(3M_1+5M_2)M_3L$	

Shear Stress

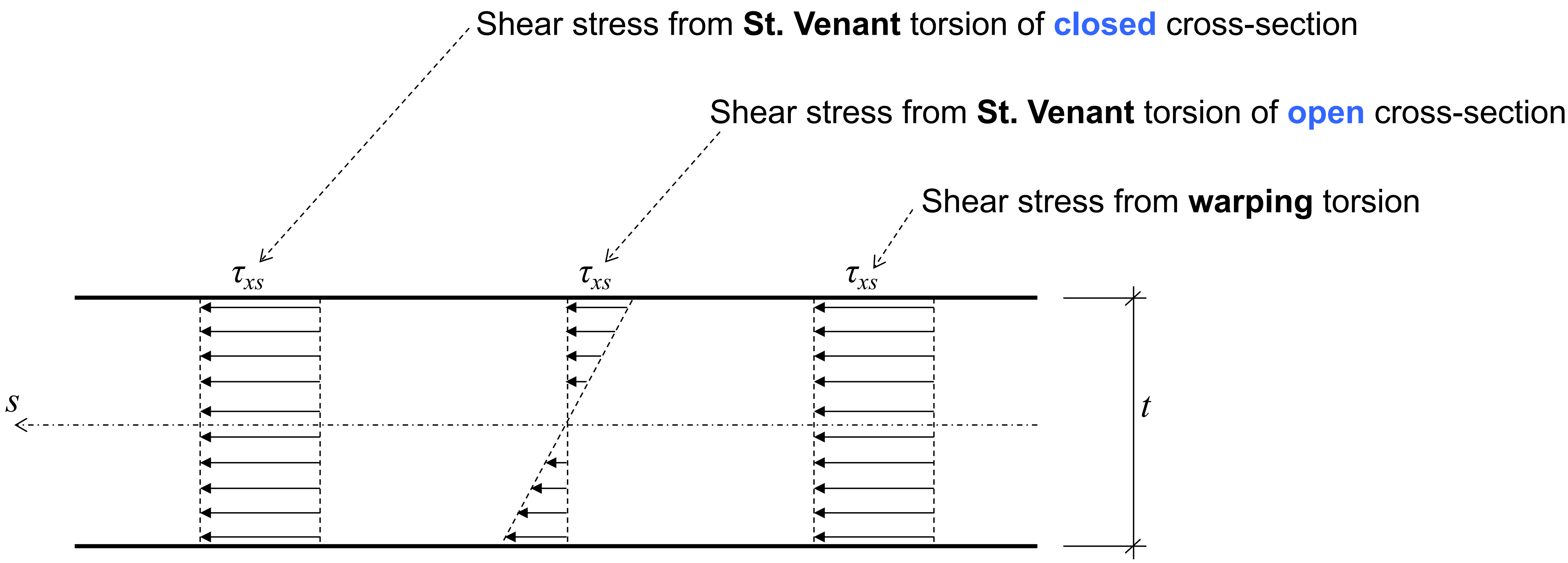


$$d\sigma_x \cdot ds \cdot t + d\tau_{xs} \cdot dx \cdot t = 0 \quad \Rightarrow \quad \frac{d\sigma_x}{dx} \cdot t + \frac{d\tau_{xs}}{ds} \cdot t = 0 \quad \Rightarrow \quad \frac{dq_s}{ds} = -\frac{d\sigma_x}{dx} \cdot t$$

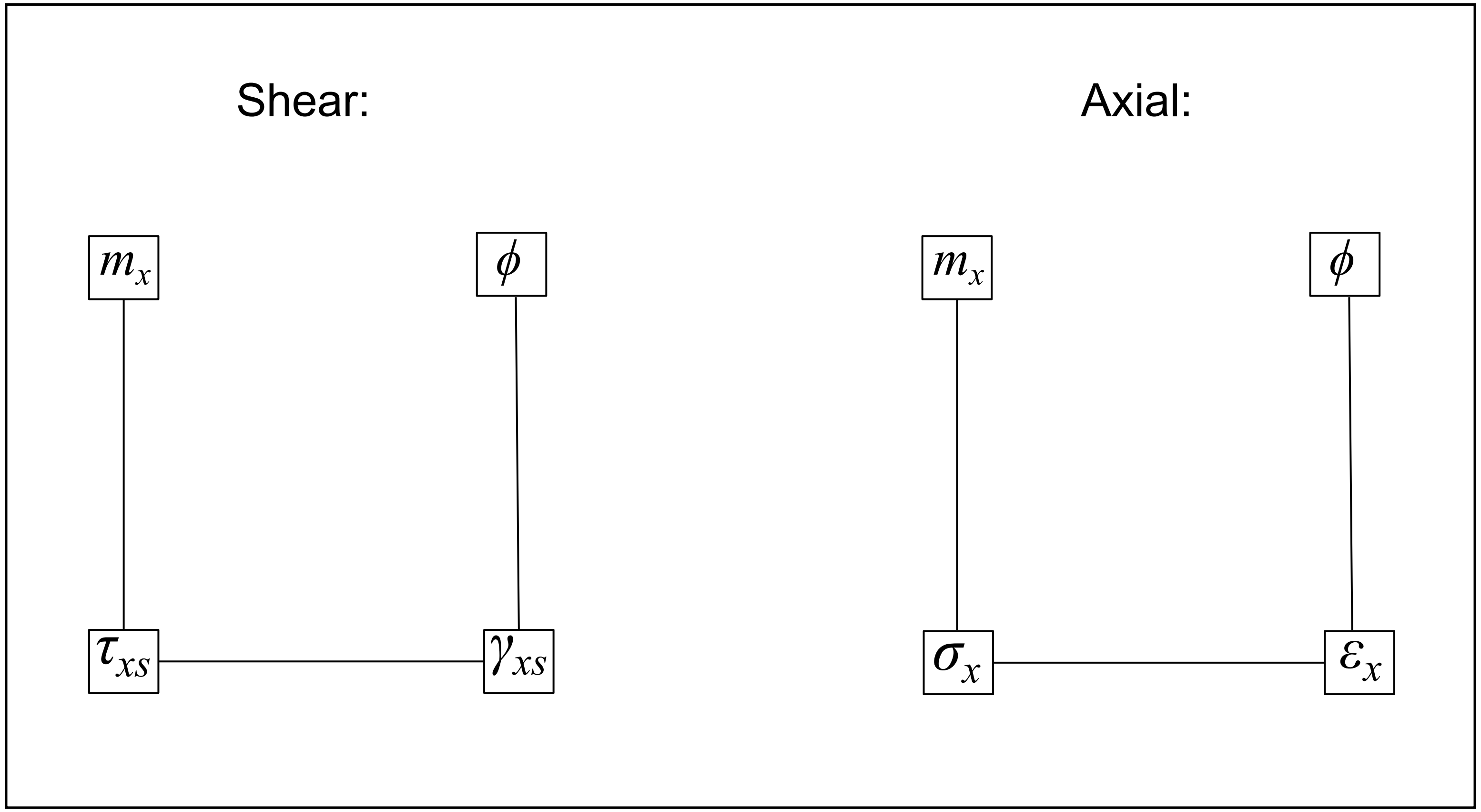
$$q_s = \int_0^s \frac{dq_s}{ds} \cdot ds = -\int_0^s t \cdot \frac{d\sigma_x}{dx} ds = -\int_0^s \frac{d\sigma_x}{dx} dA = -\int_0^s \frac{dB}{dx} \cdot \frac{1}{C_w} \cdot \Omega \, dA = -\frac{dB}{dx} \cdot \frac{1}{C_w} \cdot Q_\Omega$$

$$q_s = \frac{T}{C_w} Q_\Omega \qquad B' = -EC_w \cdot \phi''' \qquad Q_\Omega = \int_0^s \Omega \, dA = \int_0^s \Omega \cdot t \, ds$$

Shear Stress



Modified Theory



More lectures:

Terje’s Toobox:

terje.civil.ubc.ca