A short course on

Indeterminate Structures

This video:

Flexibility Method

Terje's Toolbox is freely available at terje.civil.ubc.ca
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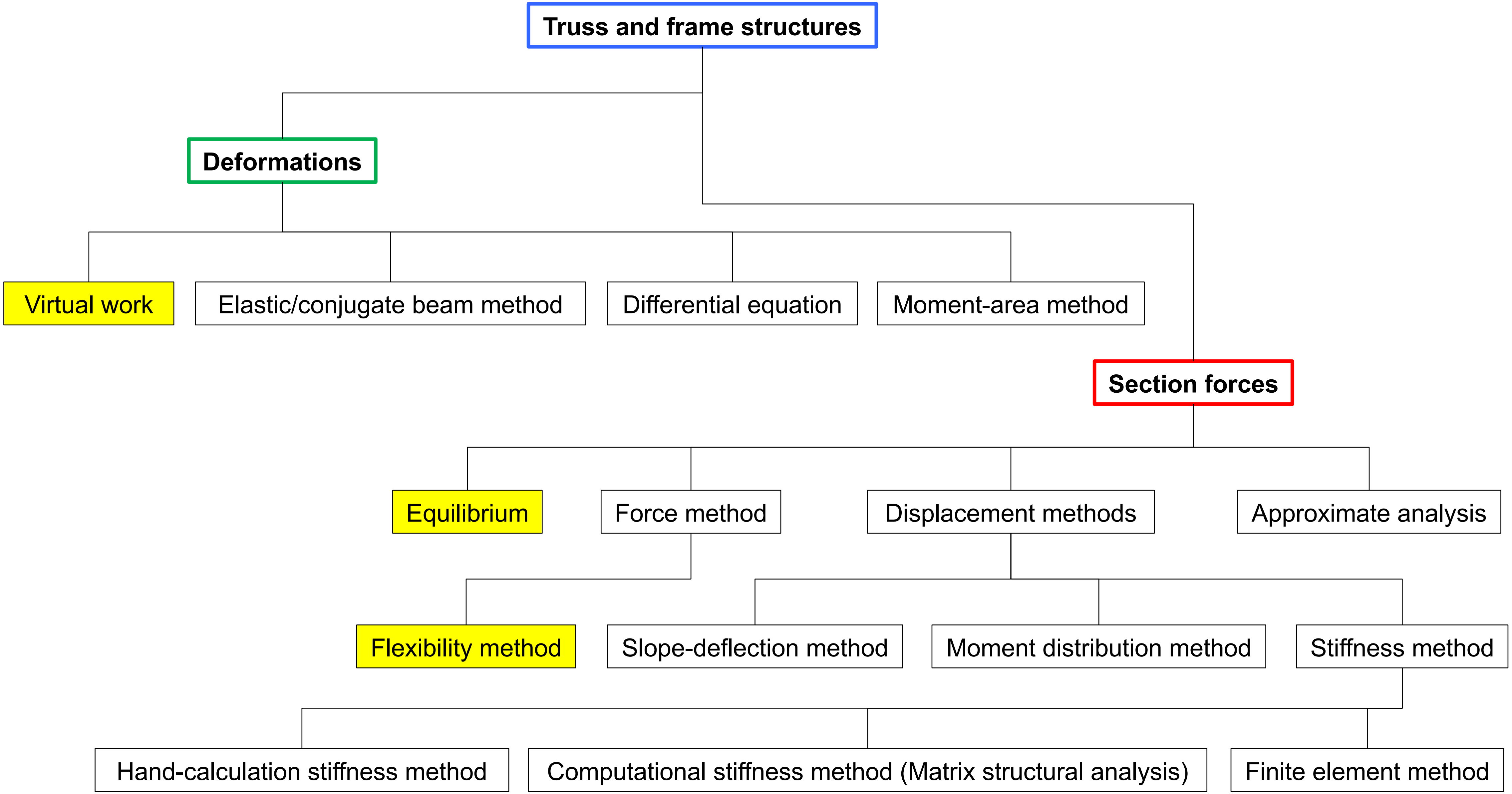
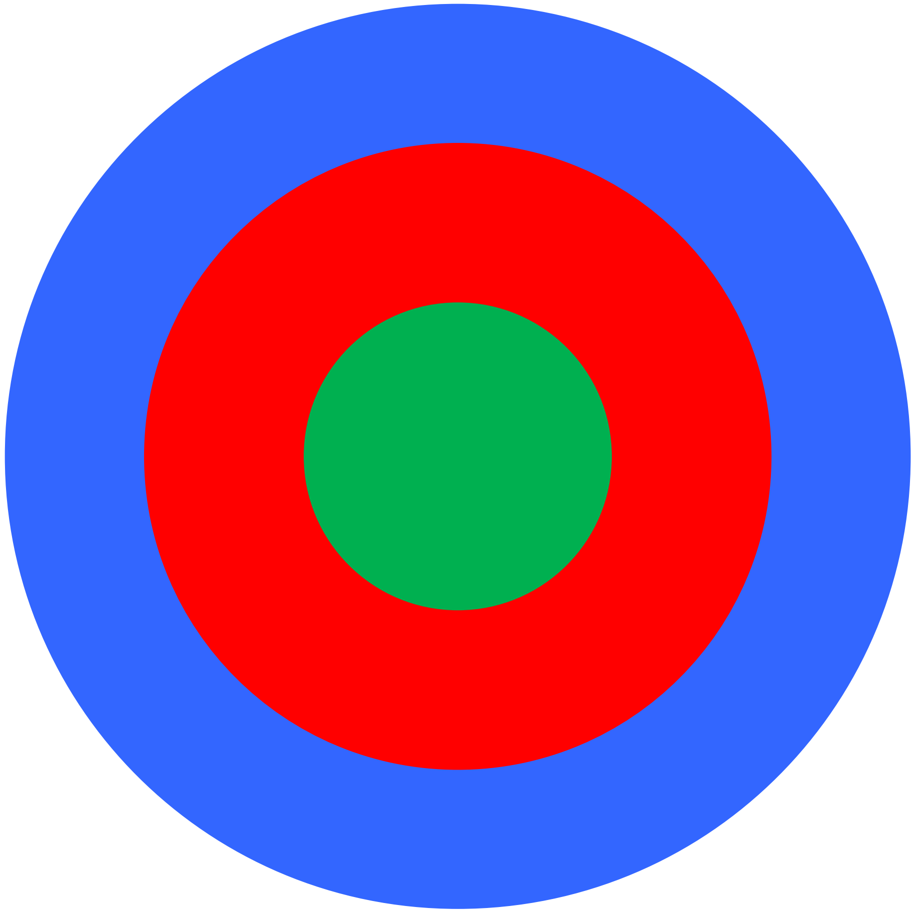
Overview of Methods



Interplay of Methods

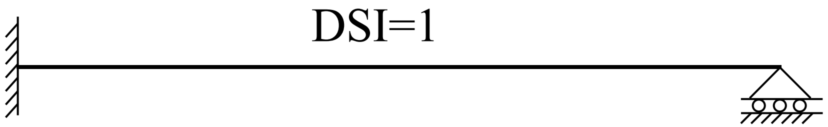


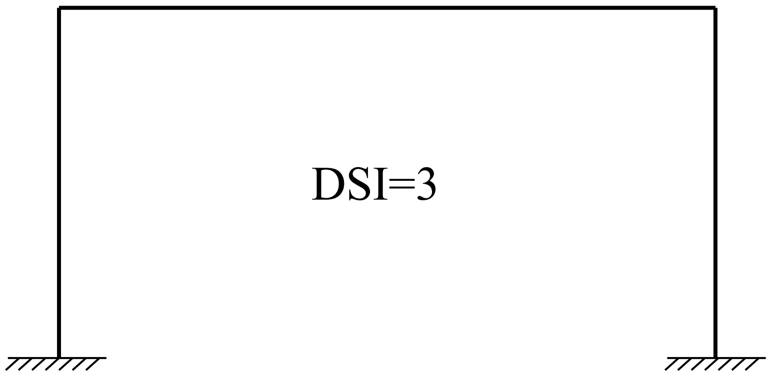
Equilibrium to find BMD and sometimes AFD for statically determinate structure

Virtual work to determine deformations

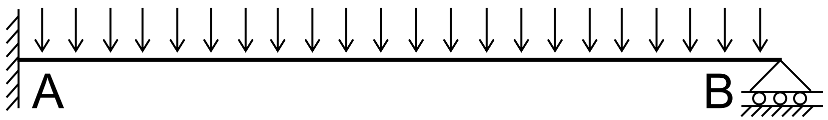
Flexibility method to determine BMD, SFD, and AFD

DSI & Redundants

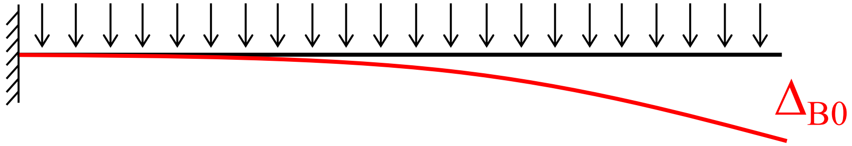




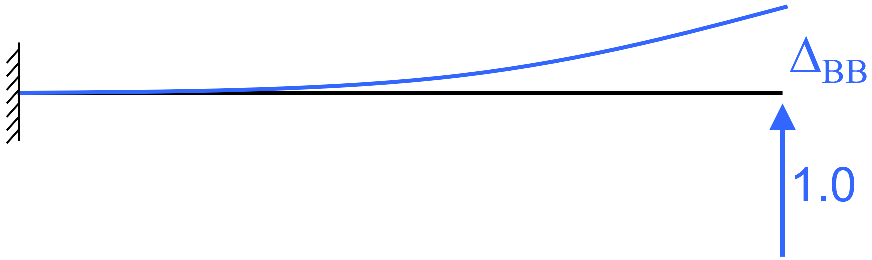
Example



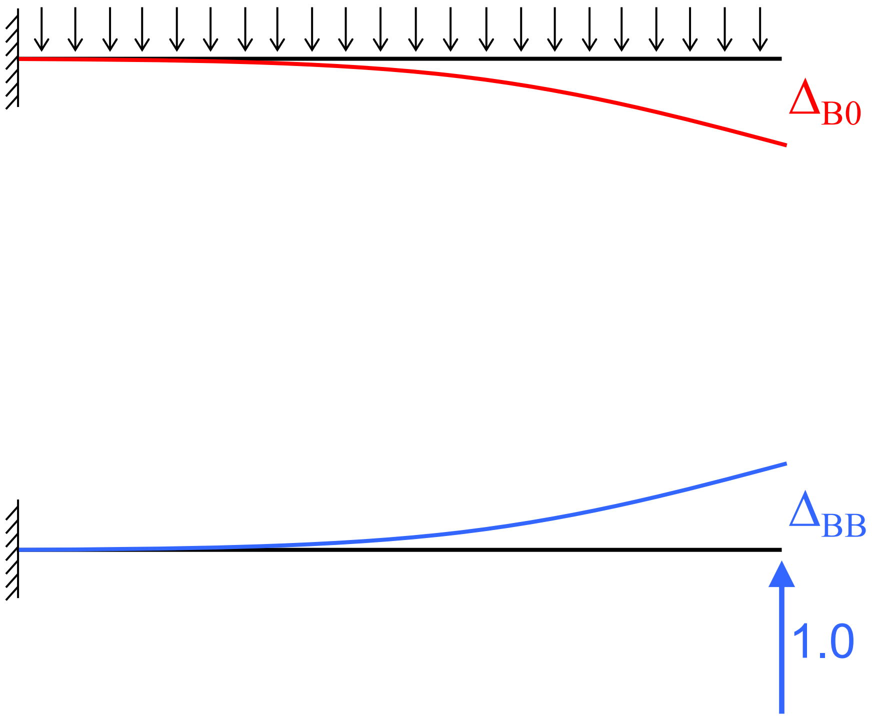
Selected redundant, X_B defines the positive direction for all Δs



Compatibility: $\Delta_{B0} + \Delta_{BB} \cdot X_B = 0$



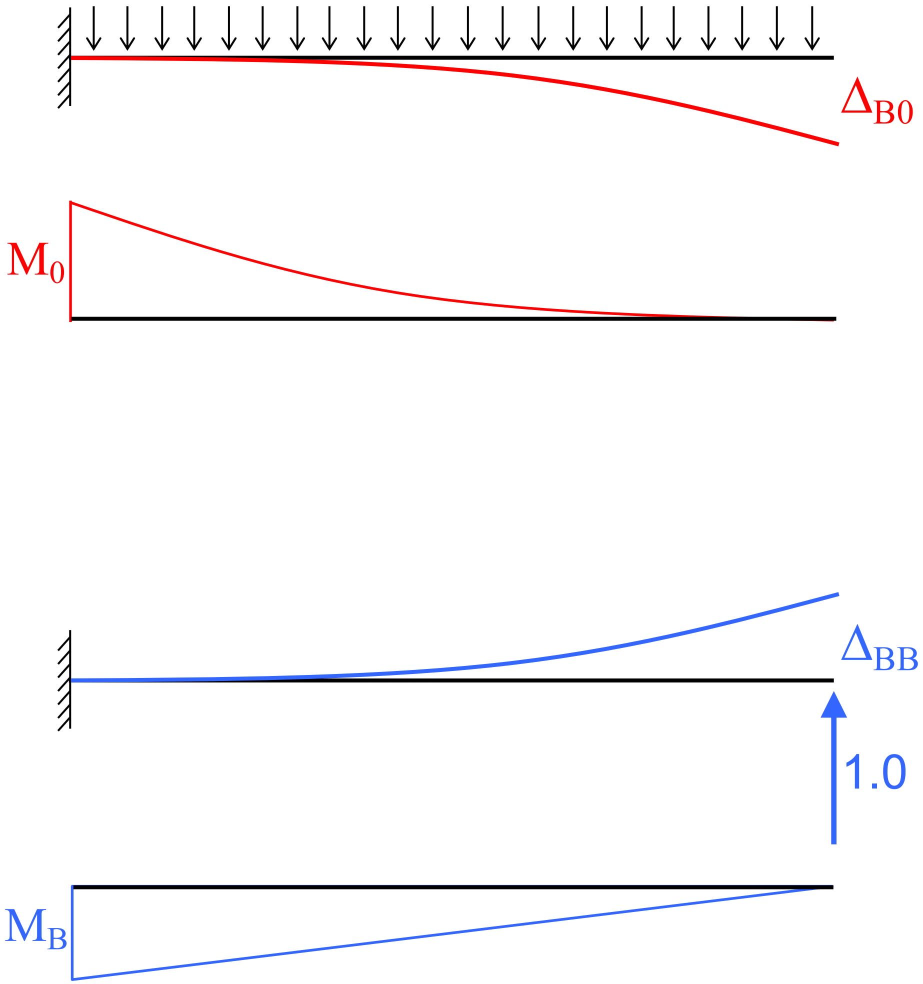
Compatibility Equation



$$\Delta_{B0} + \Delta_{BB} \cdot X_B = 0$$

$$X_B = -\frac{\Delta_{B0}}{\Delta_{BB}}$$

Virtual Work



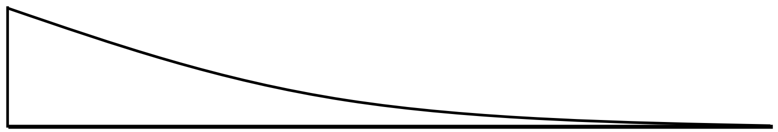
Virtual Real

$$\Delta_{B0} = \int_0^L M_B \cdot \frac{M_0}{EI}\, dx$$

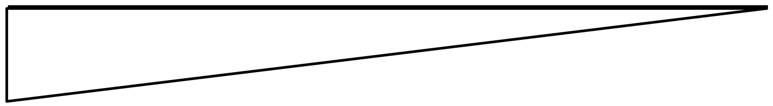
$$\Delta_{BB} = \int_0^L M_B \cdot \frac{M_B}{EI}\, dx$$

Displacements

$M_0 = \frac{qL^2}{2}$



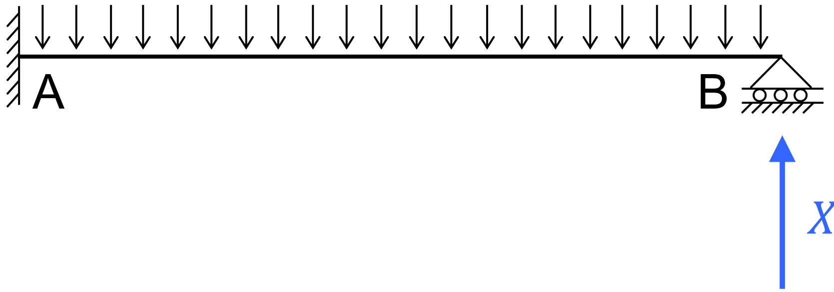
$M_B = L$



$$\Delta_{B0} = \int_0^L M_B \cdot \frac{M_0}{EI}\, dx = -\frac{1}{4EI} \cdot \frac{qL^2}{2} \cdot L \cdot L = -\frac{qL^4}{8EI}$$

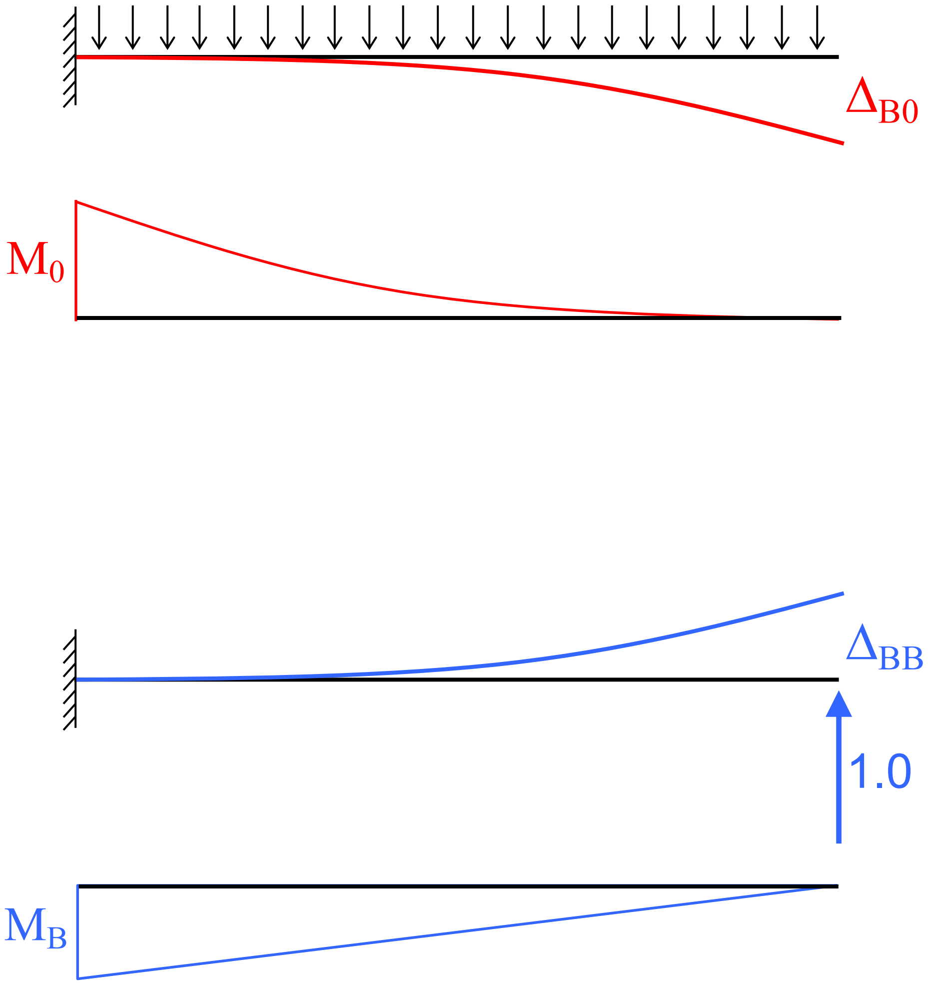
$$\Delta_{BB} = \int_0^L M_B \cdot \frac{M_B}{EI}\, dx = \frac{1}{3EI} \cdot L \cdot L \cdot L = \frac{L^3}{3EI}$$

Result



$$X_B = -\frac{\Delta_{B0}}{\Delta_{BB}} = -\frac{-\frac{qL^4}{8EI}}{\frac{L^3}{3EI}} = \frac{3qL}{8}$$

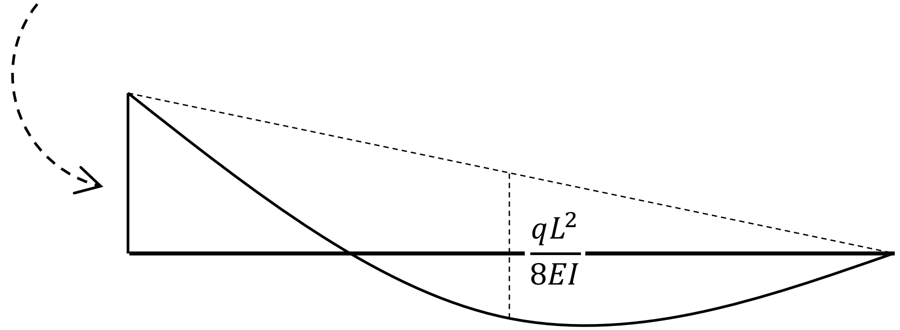
Example



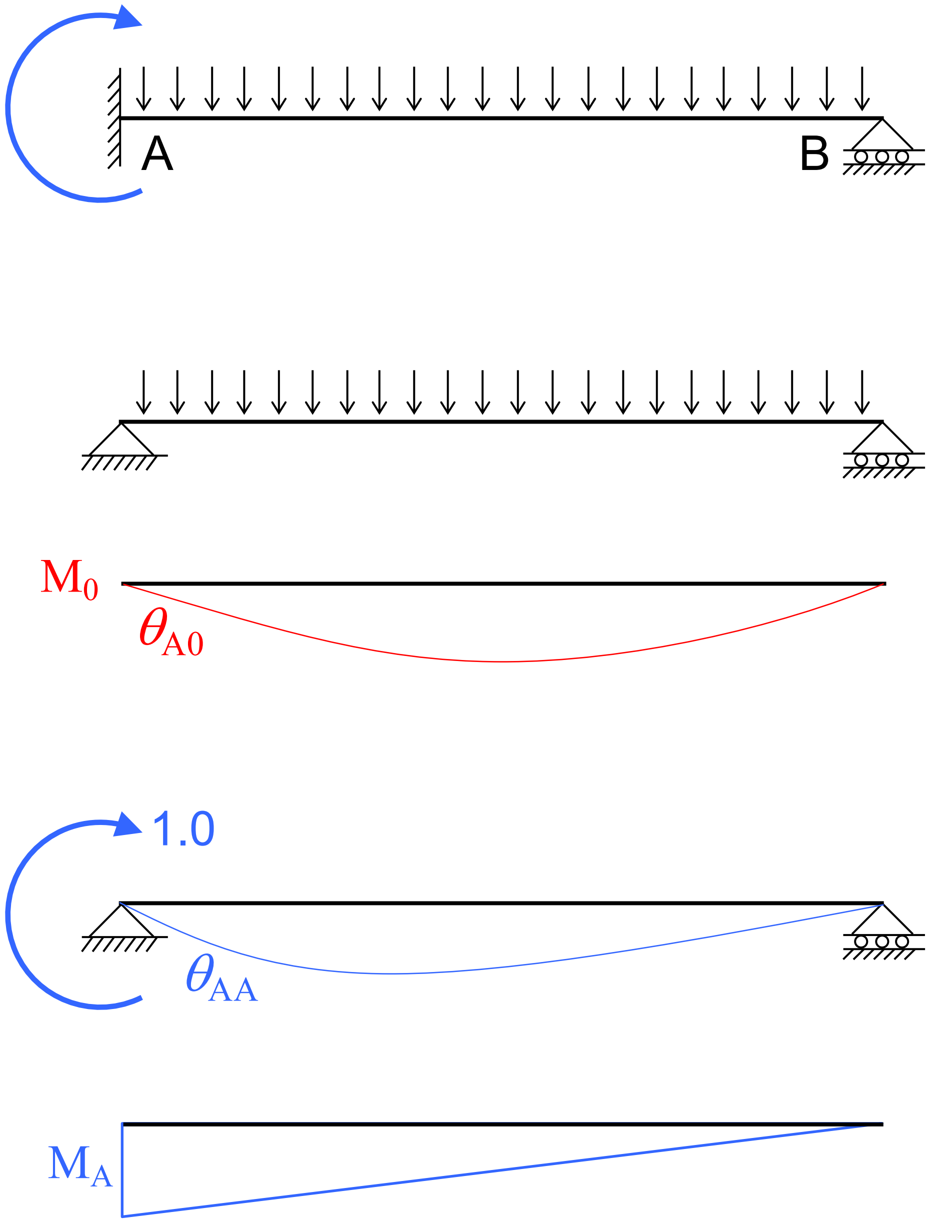
$$M = M_0 + M_B \cdot X_B$$

$$V = V_0 + V_B \cdot X_B$$

$$M = \frac{qL^2}{2} - L \cdot \frac{3qL}{8} = \frac{qL^2}{8EI}$$



Rotation



$$\theta_{A0} + \theta_{AA} \cdot X_A = 0$$

$$X_A = -\frac{\theta_{A0}}{\theta_{AA}}$$

$$M = M_0 + M_A \cdot X_A$$

$$V = V_0 + V_A \cdot X_A$$

DSI > 1

$$\Delta_{A0} + \Delta_{AA} \cdot x_A + \cdot \Delta_{AB} \cdot x_B = 0$$
$$\Delta_{B0} + \Delta_{BA} \cdot x_A + \Delta_{BB} \cdot x_B = 0$$

$$\mathbf{d} + \mathbf{f}\mathbf{x} = \mathbf{0}$$

$$M = M_0 + M_A \cdot x_A + M_B \cdot x_B$$
$$V = V_0 + V_A \cdot x_A + V_B \cdot x_B$$
$$N = N_0 + N_A \cdot x_A + N_B \cdot x_B$$

Procedure

1. Determine the degree of static indeterminacy, DSI

2. Make the structure statically determinate by introducing DSI number of releases

3. Draw BMDs for the determinate structure, for the acting load and for unit forces along the redundants

4. Establish compatibility equations to avoid “gaps” at the releases

5. Determine the deformations in the compatibility equations by virtual work

6. Solve the compatibility equations for the redundant forces

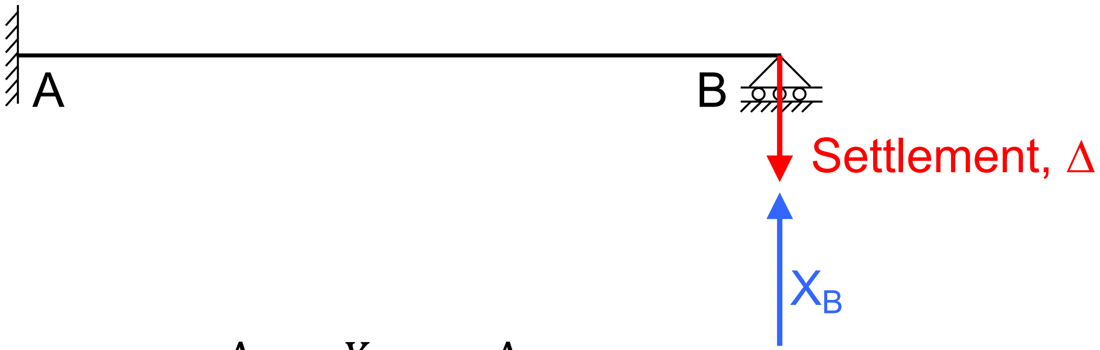
7. Draw the final BMD by summing the BMDs from Step 3 multiplied by redundant force values

Virtual Work

$$\begin{pmatrix} \delta F \cdot \Delta \\ +\delta F_{S1} \cdot \Delta_{S1} \\ +\delta F_{S2} \cdot \Delta_{S2} \\ +\cdots \end{pmatrix} = \sum_{\substack{\text{Sum over} \\ \text{all members}}} \begin{pmatrix} \delta N \cdot \left(\dfrac{N \cdot L}{EA} + \alpha \cdot \Delta T \cdot L + \Delta L_{\text{fab. error}} \right) \\ + \displaystyle\int_0^L \delta M \cdot \left(\dfrac{M}{EI} \pm \alpha \cdot \dfrac{\left| \Delta T_{top} - \Delta T_{bottom} \right|}{h} \right) dx \\ + \displaystyle\int_0^L \dfrac{\delta V \cdot V}{G \cdot A_v} \, \mathrm{d}x \end{pmatrix}$$

Settlements

Always on the left-hand side of $\Delta_{i0} + \Delta_{ii} \cdot X_i = 0$ except…



$$\Delta_{BB} \cdot X_B = -\Delta$$

Settlements & Temperature Change

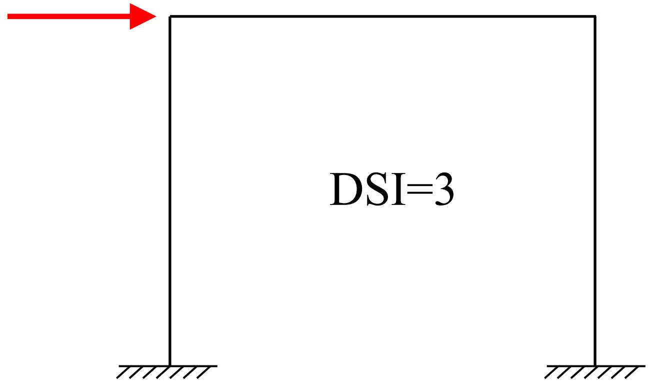
$$M = M_0 + M_B \cdot X_B$$

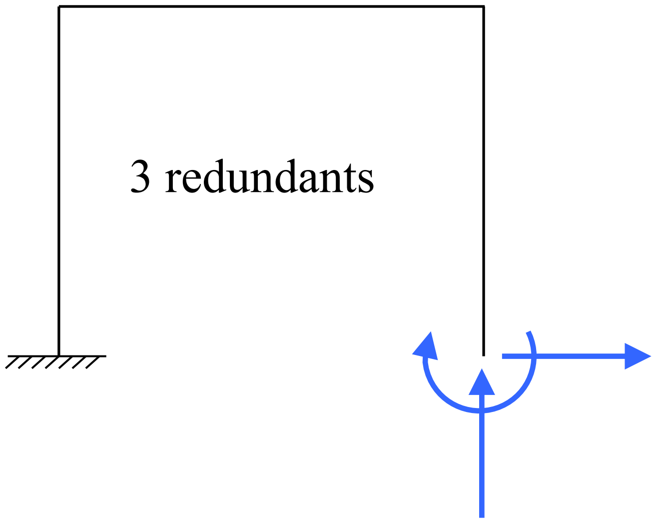
$$V = V_0 + V_B \cdot X_B$$

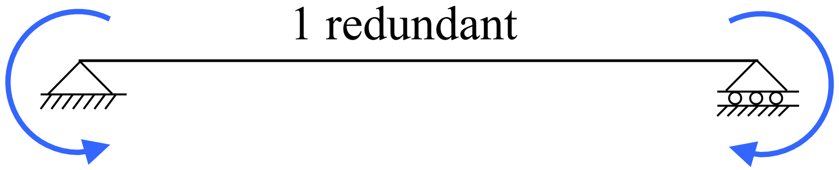
$$N = N_0 + N_B \cdot X_B$$

$$\begin{pmatrix} \delta F \cdot \Delta \\ +\delta F_{S1} \cdot \Delta_{S1} \\ +\delta F_{S2} \cdot \Delta_{S2} \\ +\cdots \end{pmatrix} = \sum_{\substack{\text{Sum over} \\ \text{all members}}} \begin{pmatrix} \delta N \cdot \left(\dfrac{N \cdot L}{EA} + \alpha \cdot \Delta T \cdot L + \Delta L_{\text{fab. error}} \right) \\ + \displaystyle\int_0^L \delta M \cdot \left(\dfrac{M}{EI} \pm \alpha \cdot \dfrac{\left| \Delta T_{top} - \Delta T_{bottom} \right|}{h} \right) dx \\ + \displaystyle\int_0^L \dfrac{\delta V \cdot V}{G \cdot A_v} \, \mathrm{d}x \end{pmatrix}$$

Always DSI Redundants?







More lectures:

Terje’s Toobox:

terje.civil.ubc.ca