A short course on

# Structural Members

This video:

**Euler-Bernoulli Beams**

Terje's Toolbox is freely available at terje.civil.ubc.ca
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# Beam Theory

Ferris wheel (1893)

Eiffel tower (1887)

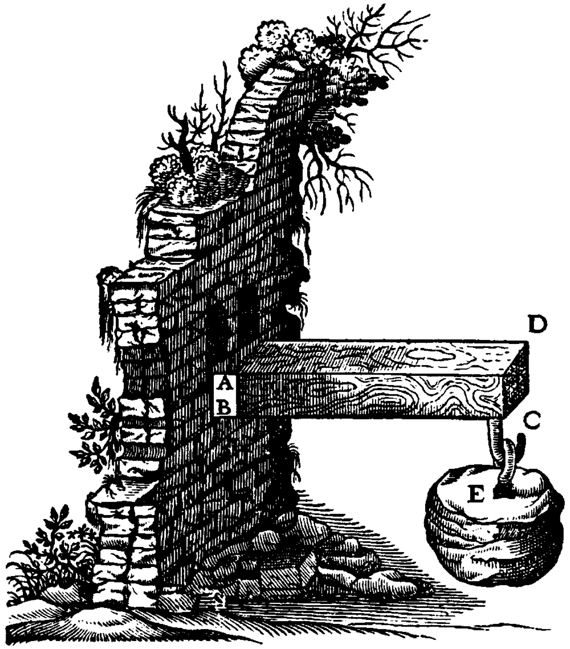
Claude-Louis Navier (1826)

Augustin Cauchy (1822)

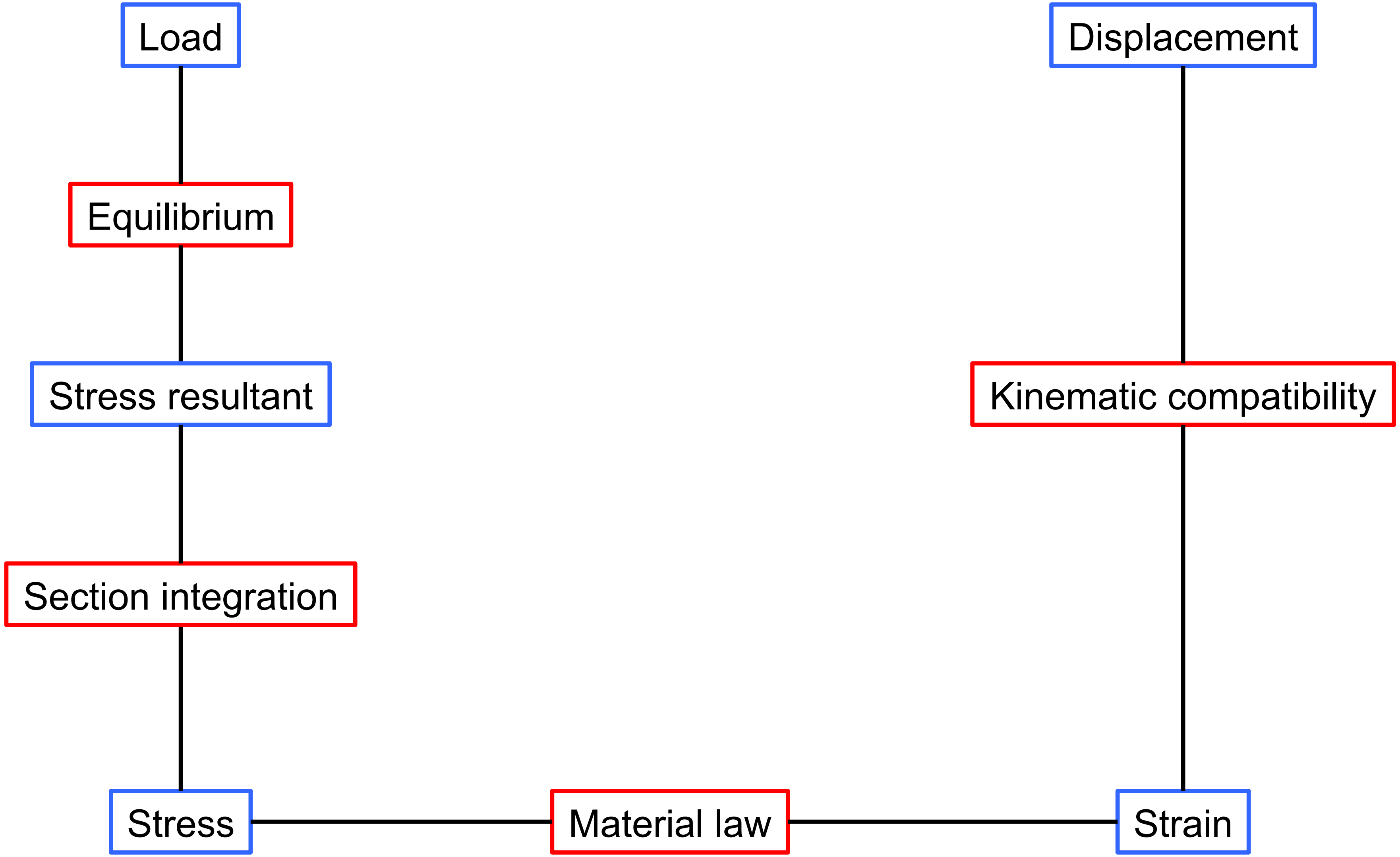
Euler-Bernoulli (1750)

Edme Mariotte (1676)

Galileo (1638)



# Ingredients



# Notation

$x$ = longitudinal direction

$z$ = vertical direction; direction of loading and displacement

$y$ = horizontal direction perpendicular to the member

$q_z$ = distributed load in the $z$-direction

$V$ = shear force, resultant of shear stress

$M$ = axial force, resultant of axial stress

$I_y$ = moment of inertia about the $y$-axis

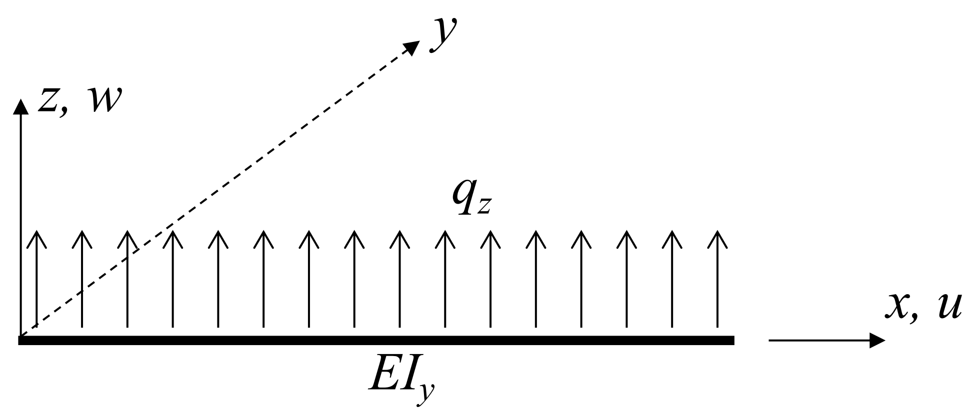
$E$ = modulus of elasticity

$\sigma$ = axial stress

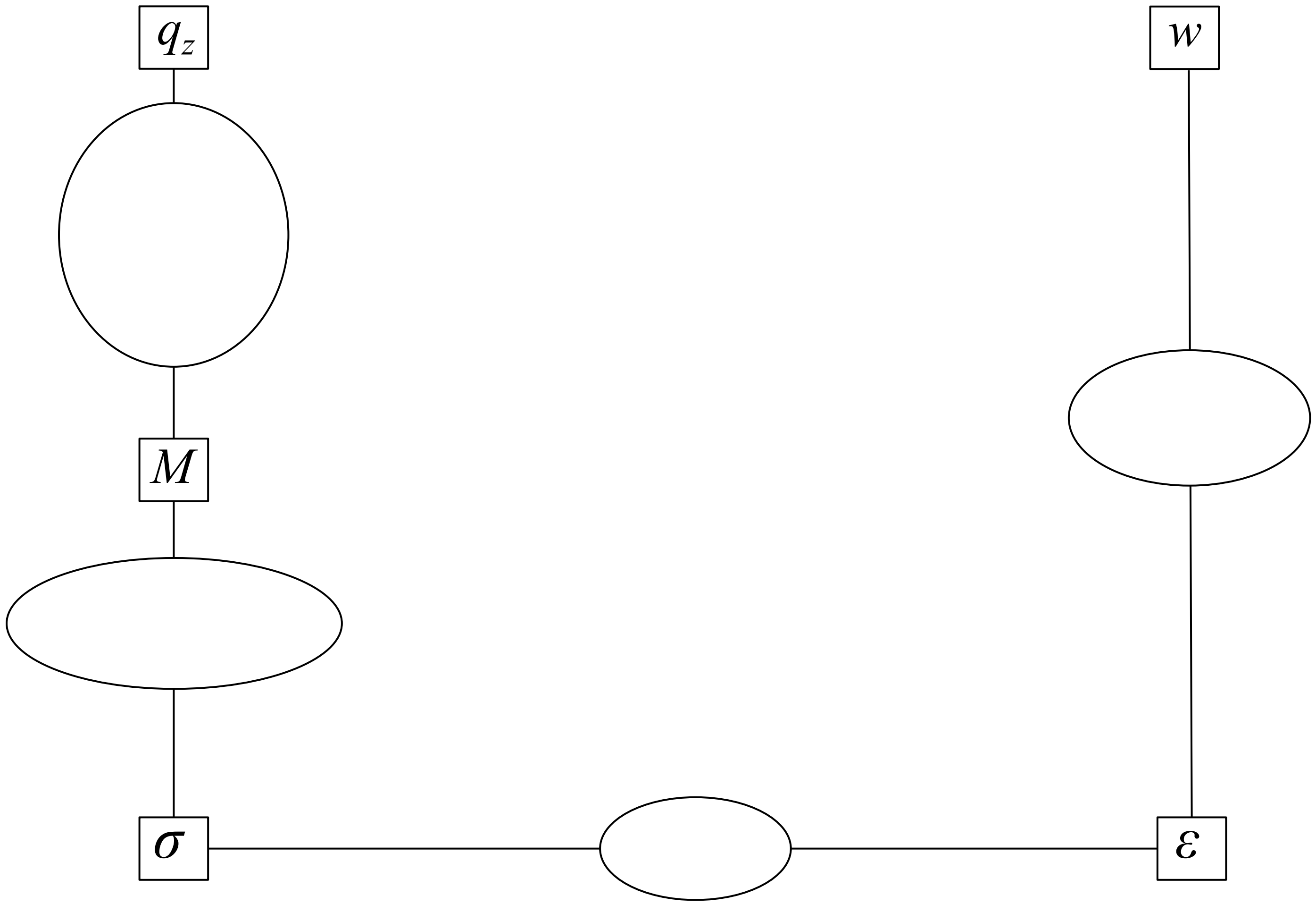
$\varepsilon$ = axial strain

$u$ = displacement in the $x$-direction

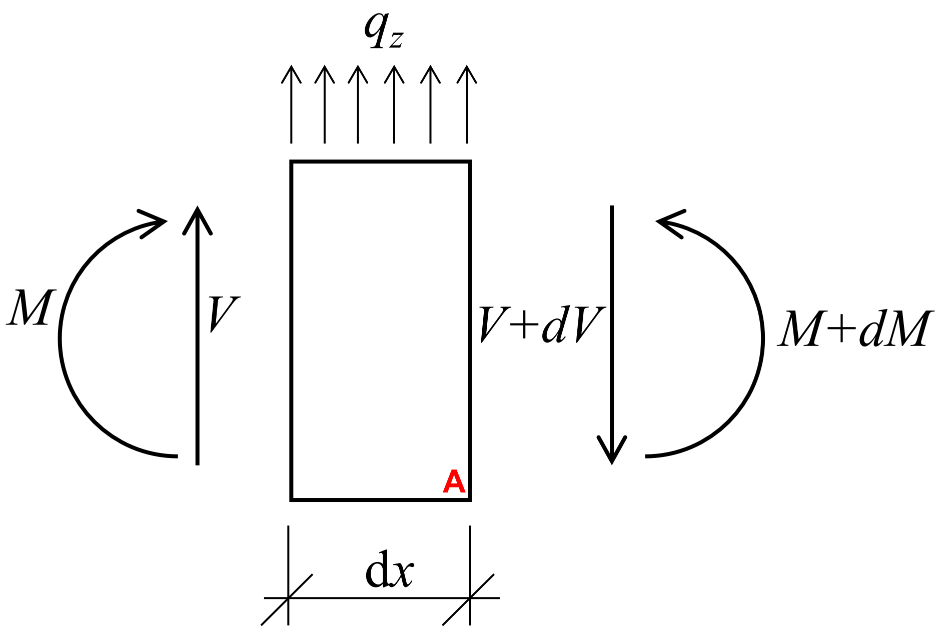
$w$ = displacement in the $z$-direction



# Anomaly



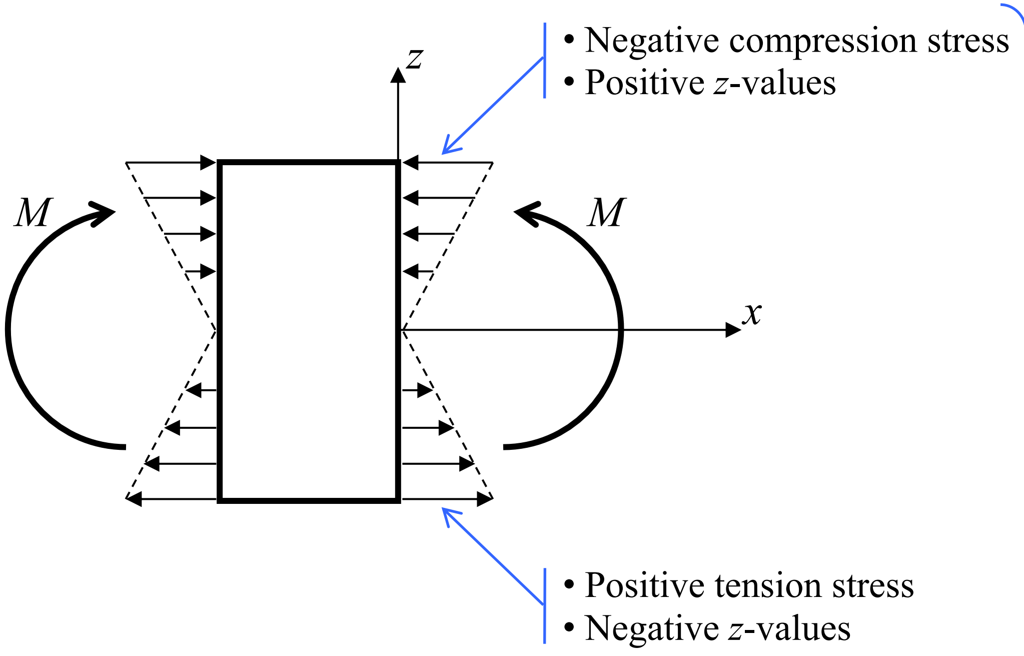
# Equilibrium




$$\Sigma F_y = V - V - dV + q_z \cdot dx = 0 \quad \Rightarrow \quad q_z = \frac{dV}{dx}$$

$$\Sigma M_A = V \cdot dx + M - M - dM + q_z \cdot dx \cdot (dx/2) = 0 \quad \Rightarrow \quad V = \frac{dM}{dx}$$

# Section Integration




Minus sign in cross-section integral is necessary to get positive bending moment

$$M = \int_A -\sigma \cdot z \; \mathrm{d}A$$

# Material Law

$$\sigma = E \cdot \varepsilon$$

$$\varepsilon_{yy} = \frac{\sigma_{yy}}{E} - \nu \cdot \frac{\sigma_{xx}}{E} = 0 \qquad \Rightarrow \qquad \sigma_{yy} = \nu \cdot \sigma_{xx}$$

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \nu \frac{\sigma_{yy}}{E} = \frac{\sigma_{xx}}{E} - \nu \frac{(\nu \cdot \sigma_{xx})}{E} = \frac{\sigma_{xx}}{E}(1-\nu^2) \qquad \Rightarrow \qquad \sigma_{xx} = \frac{E}{1-\nu^2} \cdot \varepsilon_{xx}$$

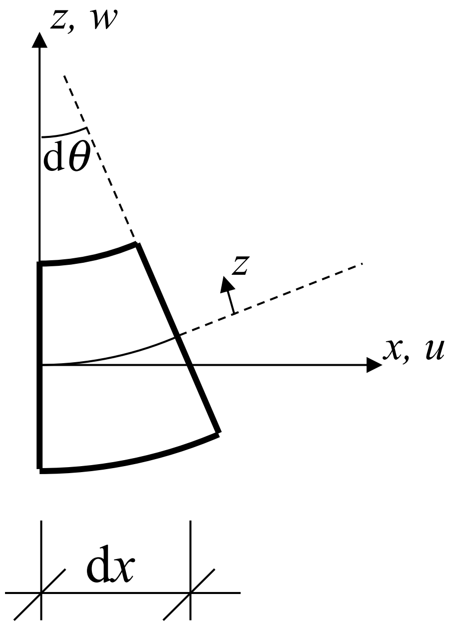
# Kinematic Compatibility

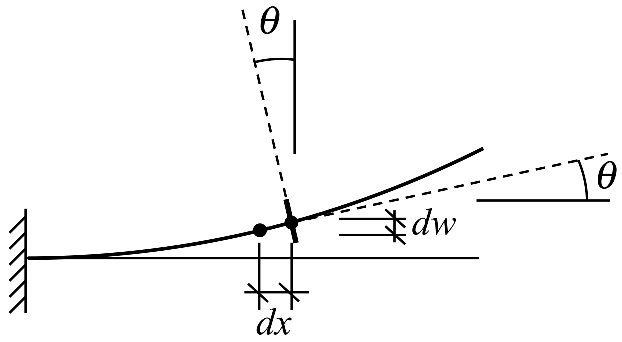
$$\varepsilon = \frac{du}{dx}$$

$$du = -\mathrm{d}\theta \cdot z$$

$$\tan(\theta) = \frac{dw}{dx} \approx \theta$$

$$\varepsilon = \frac{du}{dx} = -\frac{d\theta}{dx} \cdot z = -\frac{d^2 w}{dx^2} \cdot z$$




$$\kappa \equiv \frac{1}{R}$$

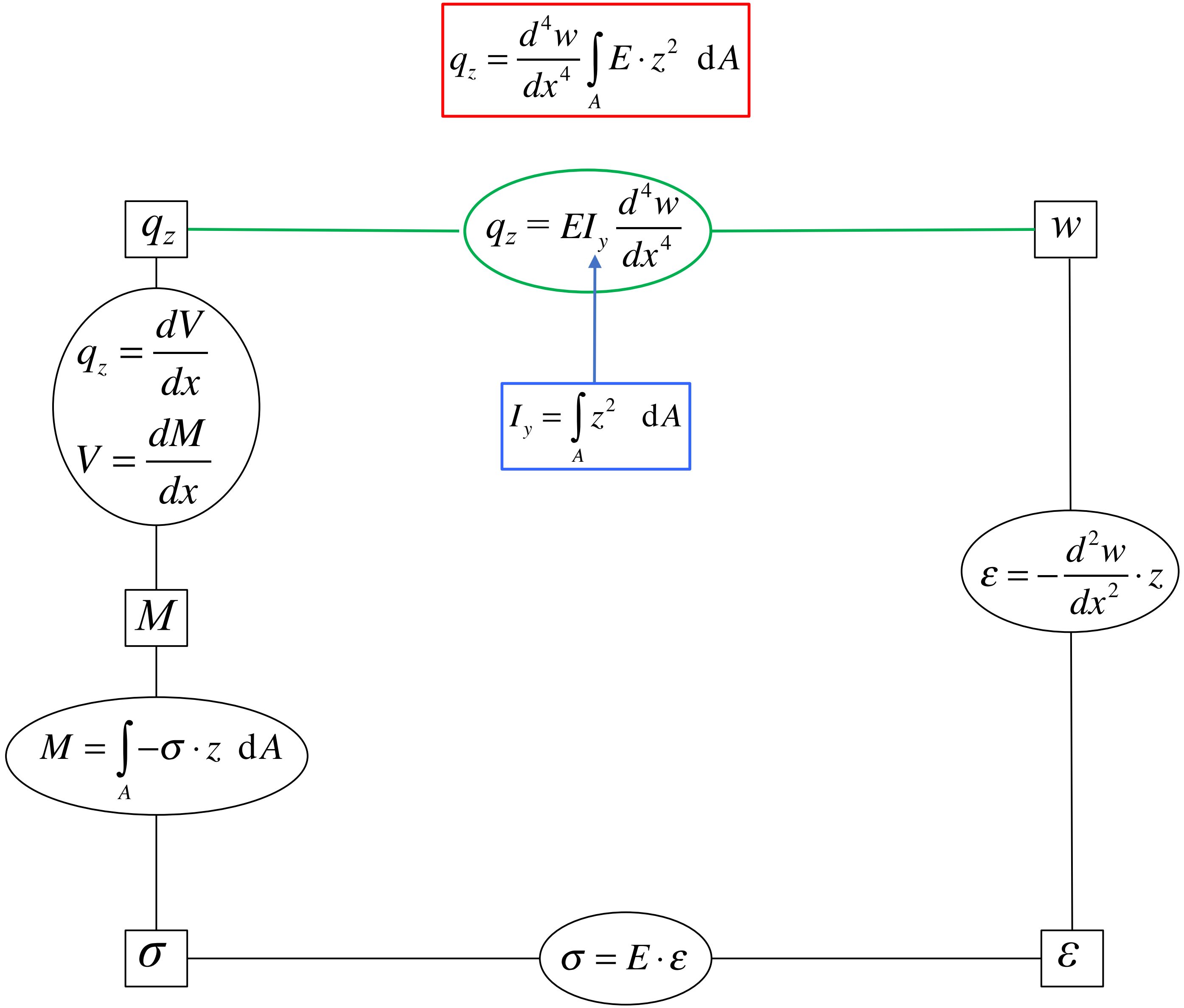
$$\kappa \approx \frac{d\theta}{dx} \approx \frac{d^2 w}{dx^2}$$

$$\theta = \tan^{-1}\left(\frac{dw}{dx}\right)$$

$$\kappa \approx \frac{d\theta}{dx} = \frac{\left(\frac{d^2 w}{dx^2}\right)}{\left(1 + \left(\frac{dw}{dx}\right)^2\right)}$$

$$\kappa = \frac{\left(\frac{d^2 w}{dx^2}\right)}{\left(1 + \left(\frac{dw}{dx}\right)^2\right)^{\frac{3}{2}}}$$

# Summary


$q_z = \frac{d^4 w}{dx^4} \int_A E \cdot z^2 \;\mathrm{d}A$
$q_z$
$q_z = EI_y \frac{d^4 w}{dx^4}$
$w$
$q_z = \frac{dV}{dx}$
$V = \frac{dM}{dx}$
$I_y = \int_A z^2 \;\mathrm{d}A$
$\varepsilon = -\frac{d^2 w}{dx^2} \cdot z$
$M$
$M = \int_A -\sigma \cdot z \;\mathrm{d}A$
$\sigma$
$\sigma = E \cdot \varepsilon$
$\varepsilon$

# General Solution

$$q_z = EI_y \frac{d^4 w}{dx^4} \longrightarrow w(x) = \frac{1}{24} \cdot \frac{q_z}{EI_y} \cdot x^4 + C_1 \cdot x^3 + C_2 \cdot x^2 + C_3 \cdot x + C_4$$

$$\theta = \frac{dw}{dx}$$

$$M = EI_y \frac{d^2 w}{dx^2}$$

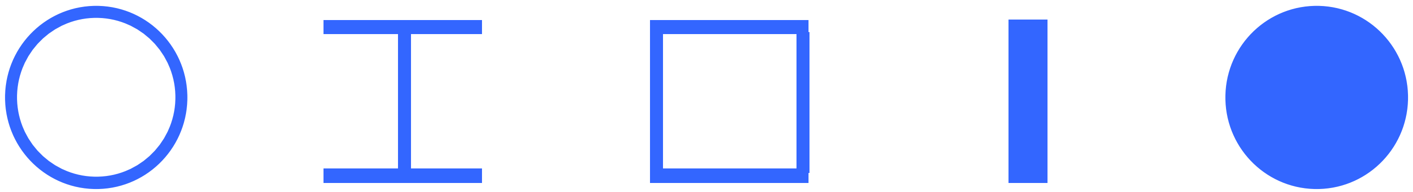
$$V = EI_y \frac{d^3 w}{dx^3}$$

# Cross-section Analysis

Moment of inertia

Axial stress

Shear stress



More lectures:

Terje’s Toobox:

terje.civil.ubc.ca