A short course on

Cross-section Analysis

This video:

Shear Centre, Shear Flow, and Shear Stress from Bending

Terje's Toolbox is freely available at terje.civil.ubc.ca
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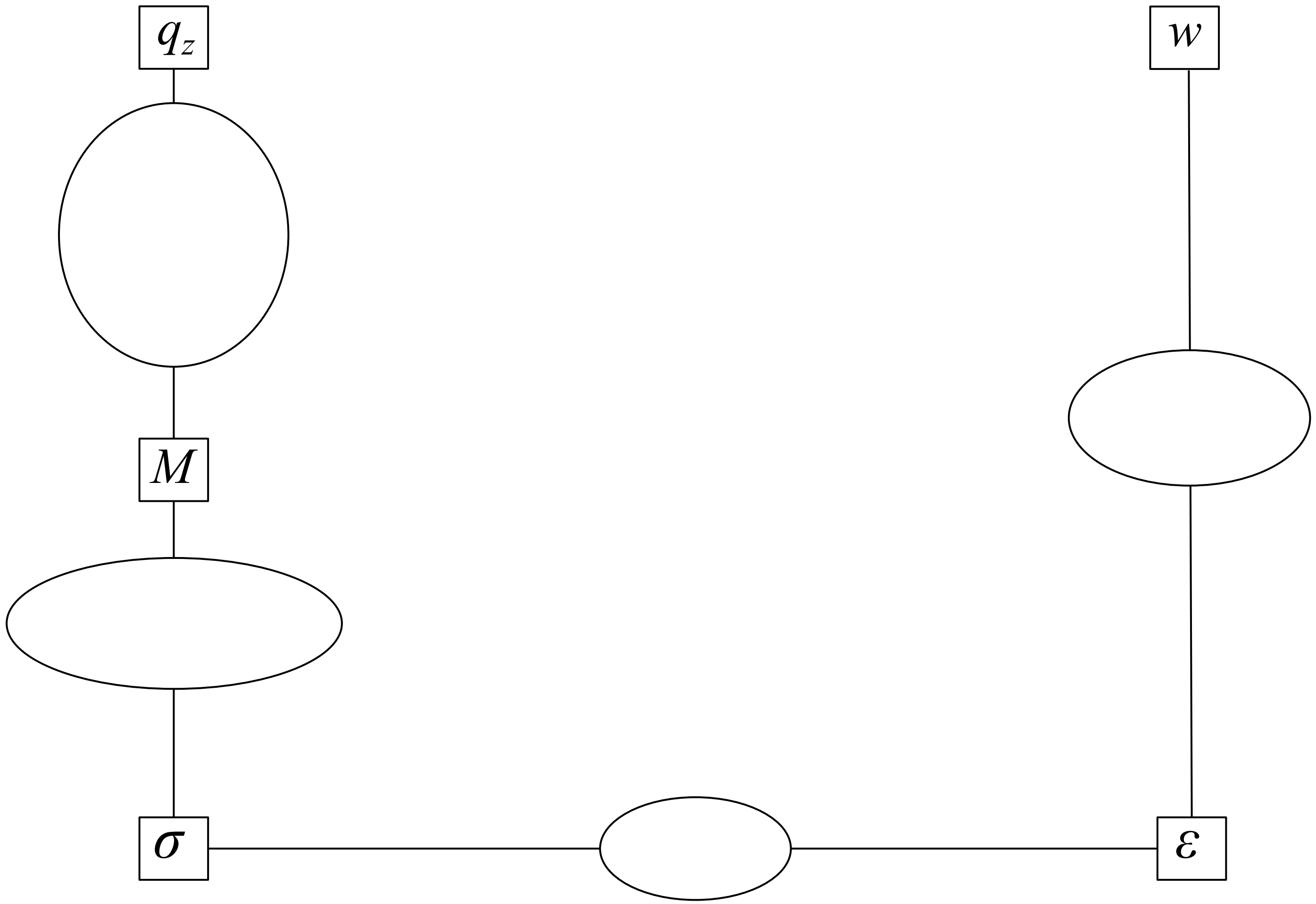
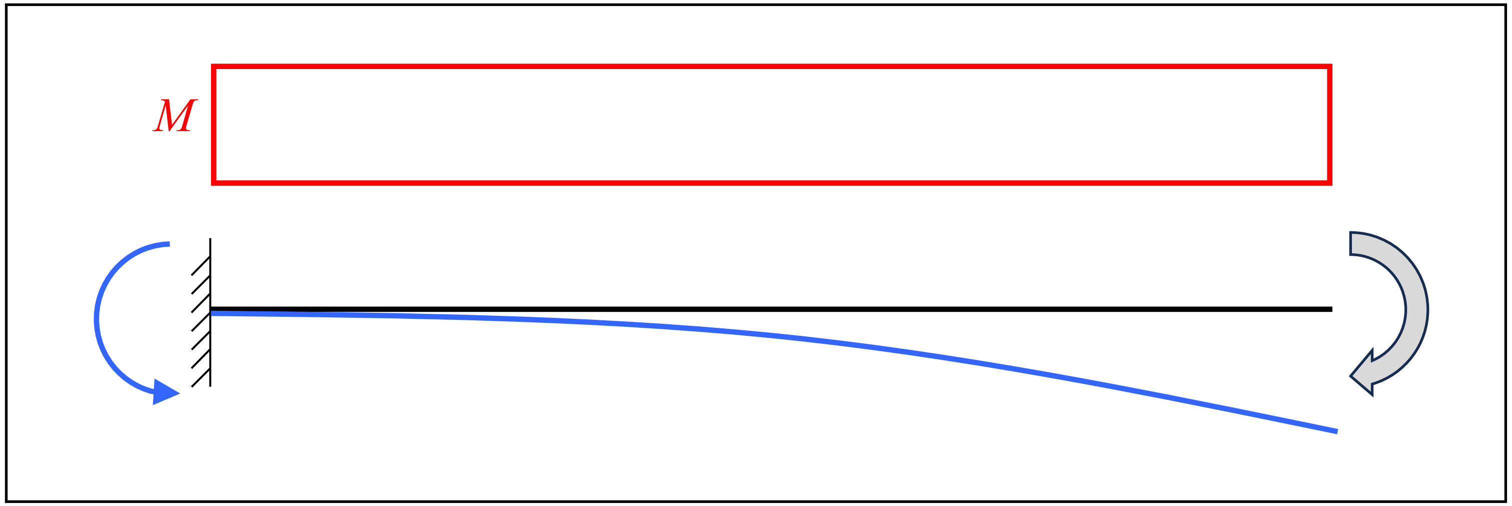
Scope

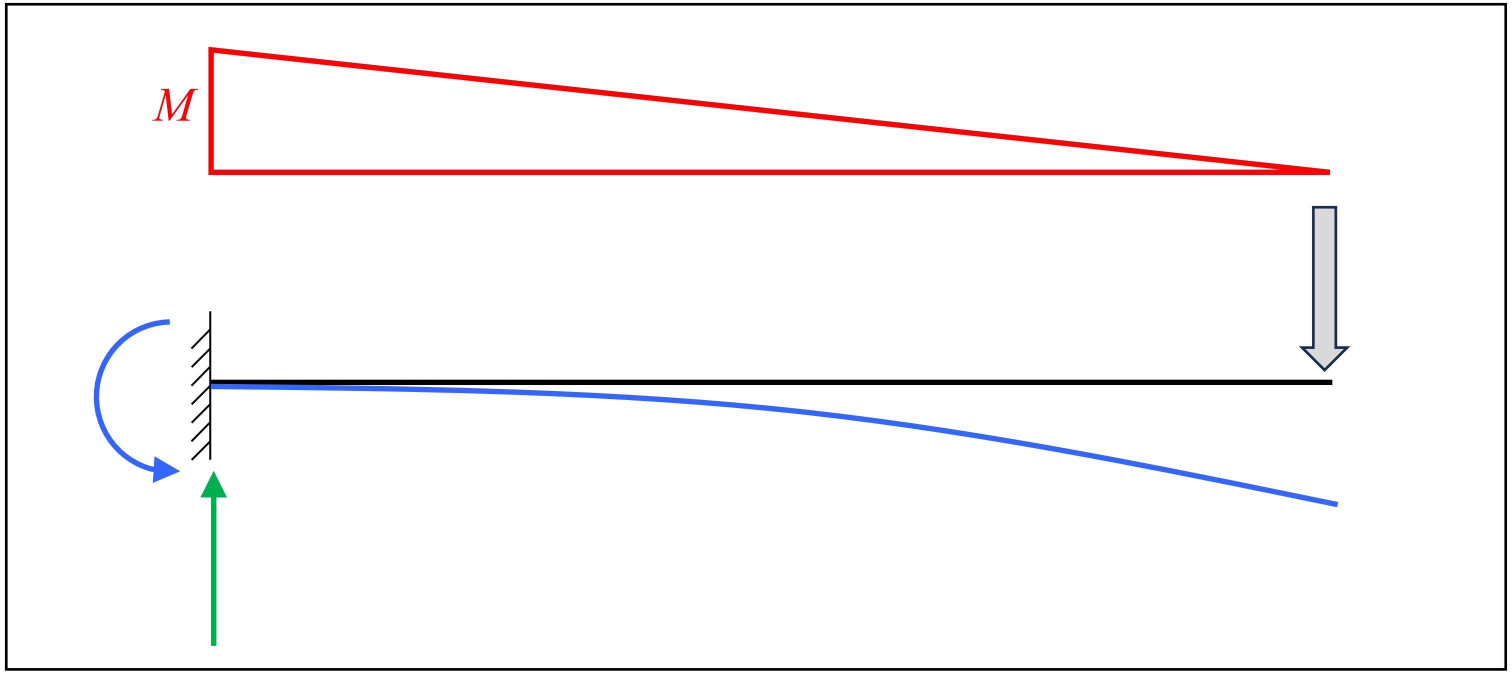
y_o, z_o	=	centroid coordinates
y_{sc}, z_{sc}	=	shear centre coordinates
A	=	cross-section area
I_y , I_z	=	moments of inertia
I_{yz}	=	product of inertia
θ	=	orientation of principal axes
J	=	Saint Venant torsion constant
Ω	=	omega diagram
C_w	=	warping torsion constant
A_{vy}, A_{vz}	=	shear area
σ	=	axial stress
τ	=	shear stress
q_s	=	shear flow

Anomaly

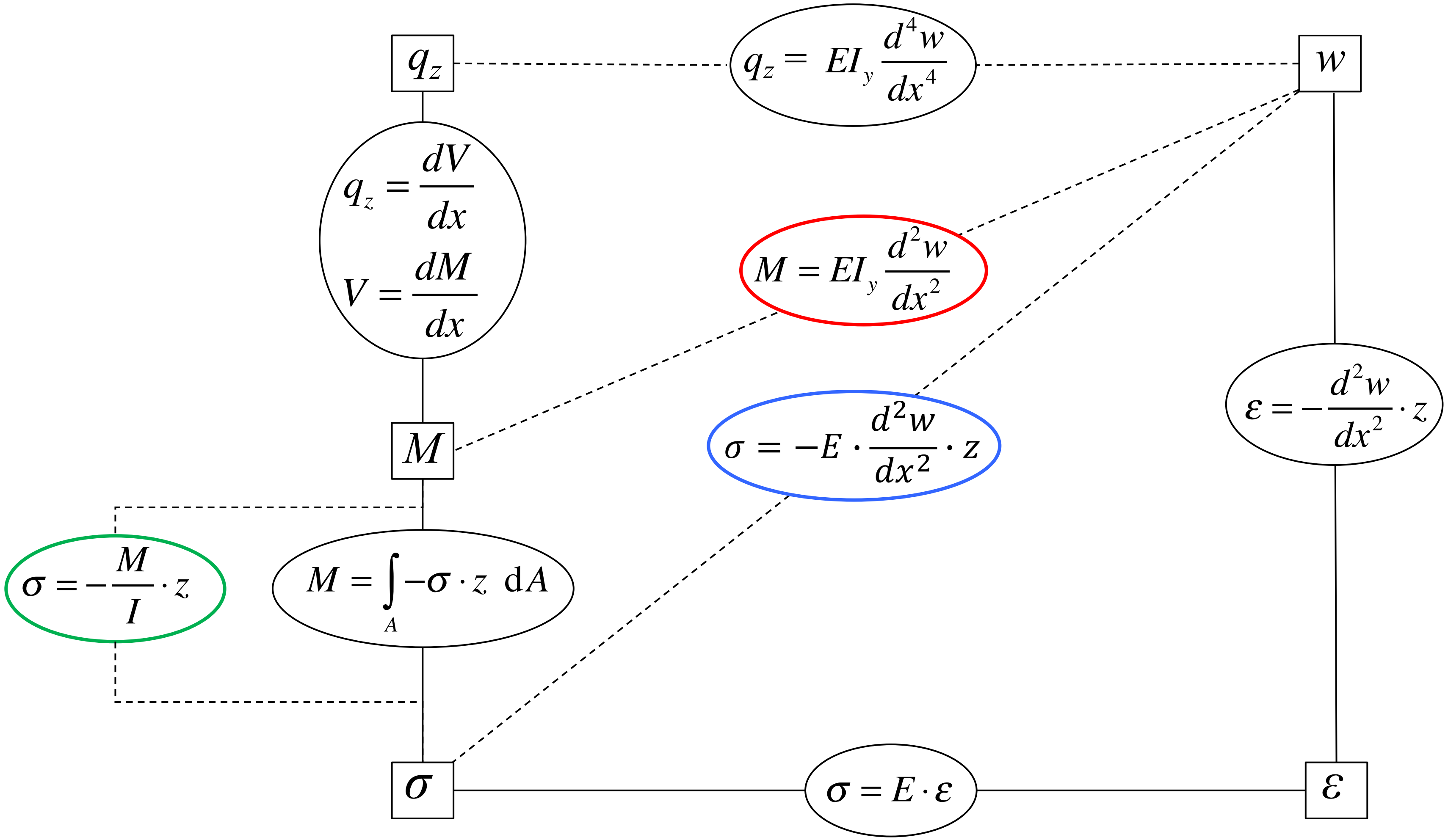


Shear is from Change in Moment

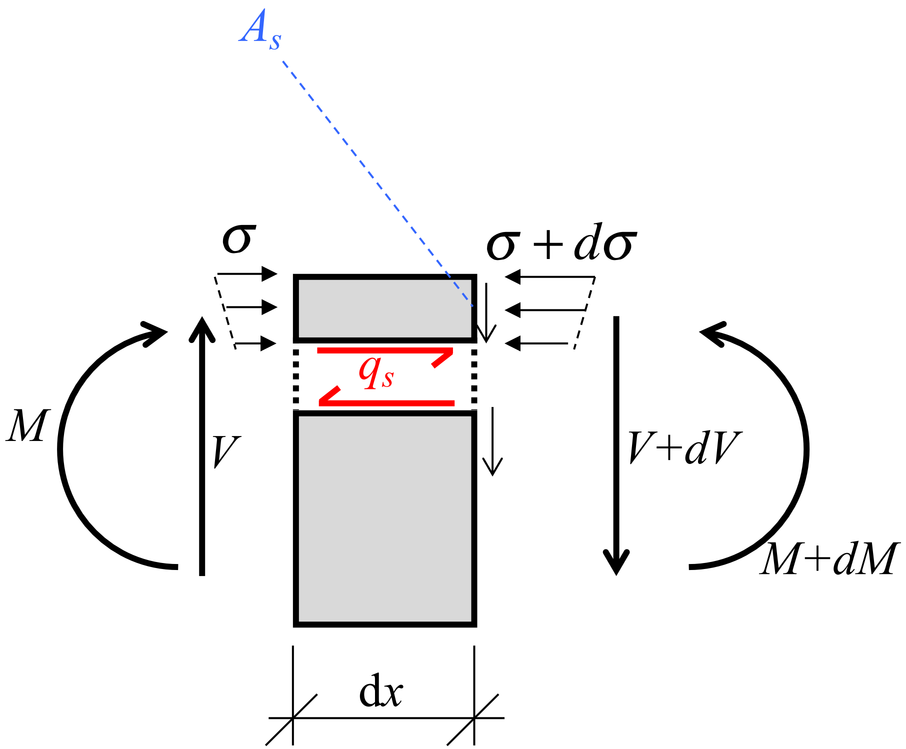




Axial Stress



Shear Flow



$$q_s \cdot dx = \int_{A_s} d\sigma \, dA = \int_{A_s} \frac{dM}{I} \cdot z \, dA$$

$$q_s = \frac{V}{I} \cdot Q$$

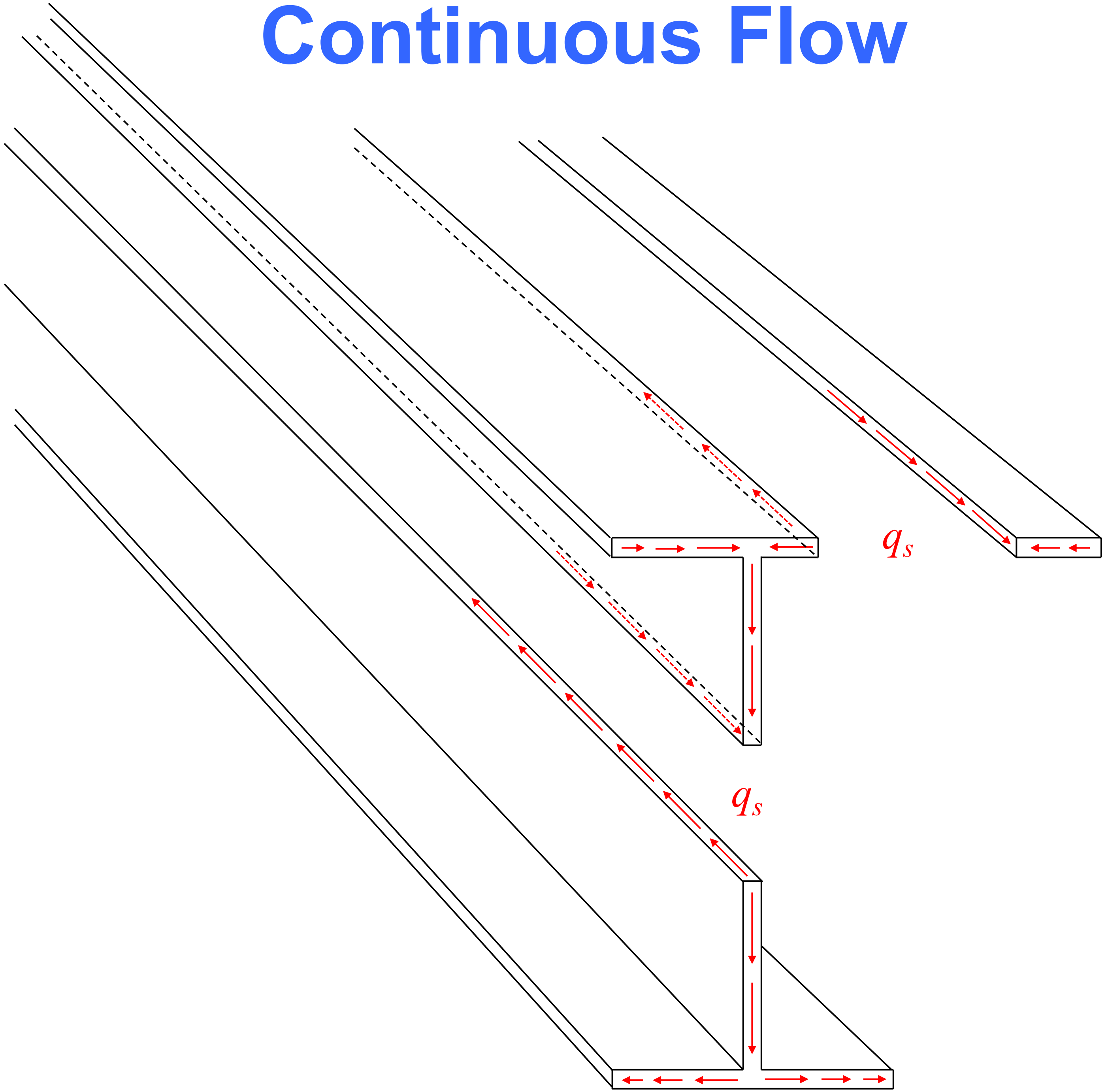
$$\boxed{Q = \int_{A_s} z \, dA}$$

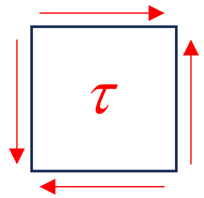
$$\Sigma M = V \cdot dx + M - M - dM = 0$$

$$Q = \sum_{i=1}^{N} z_i A_i$$

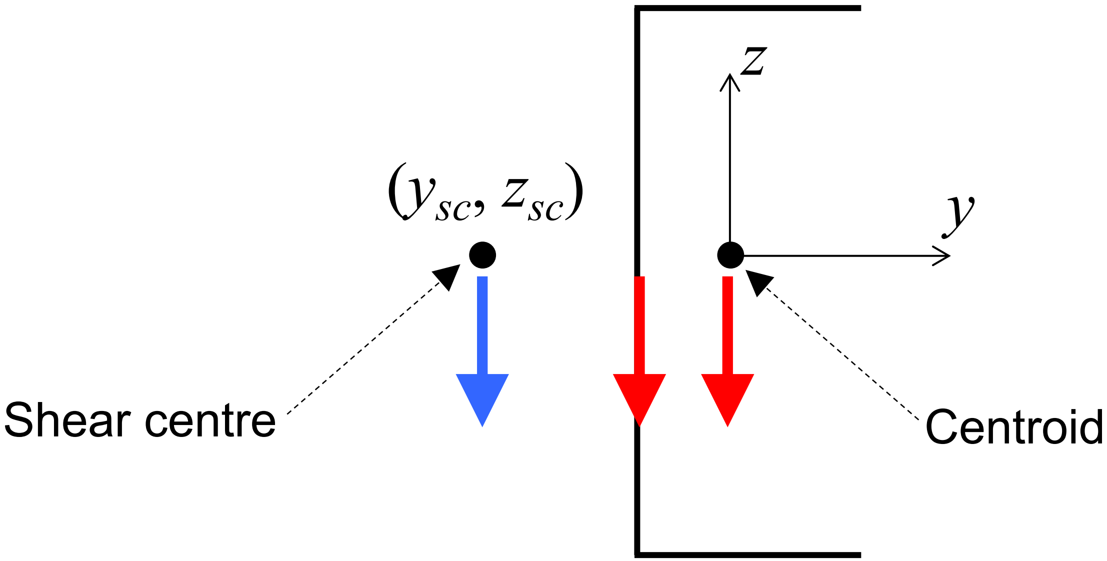
$$V = \frac{dM}{dx}$$

Continuous Flow

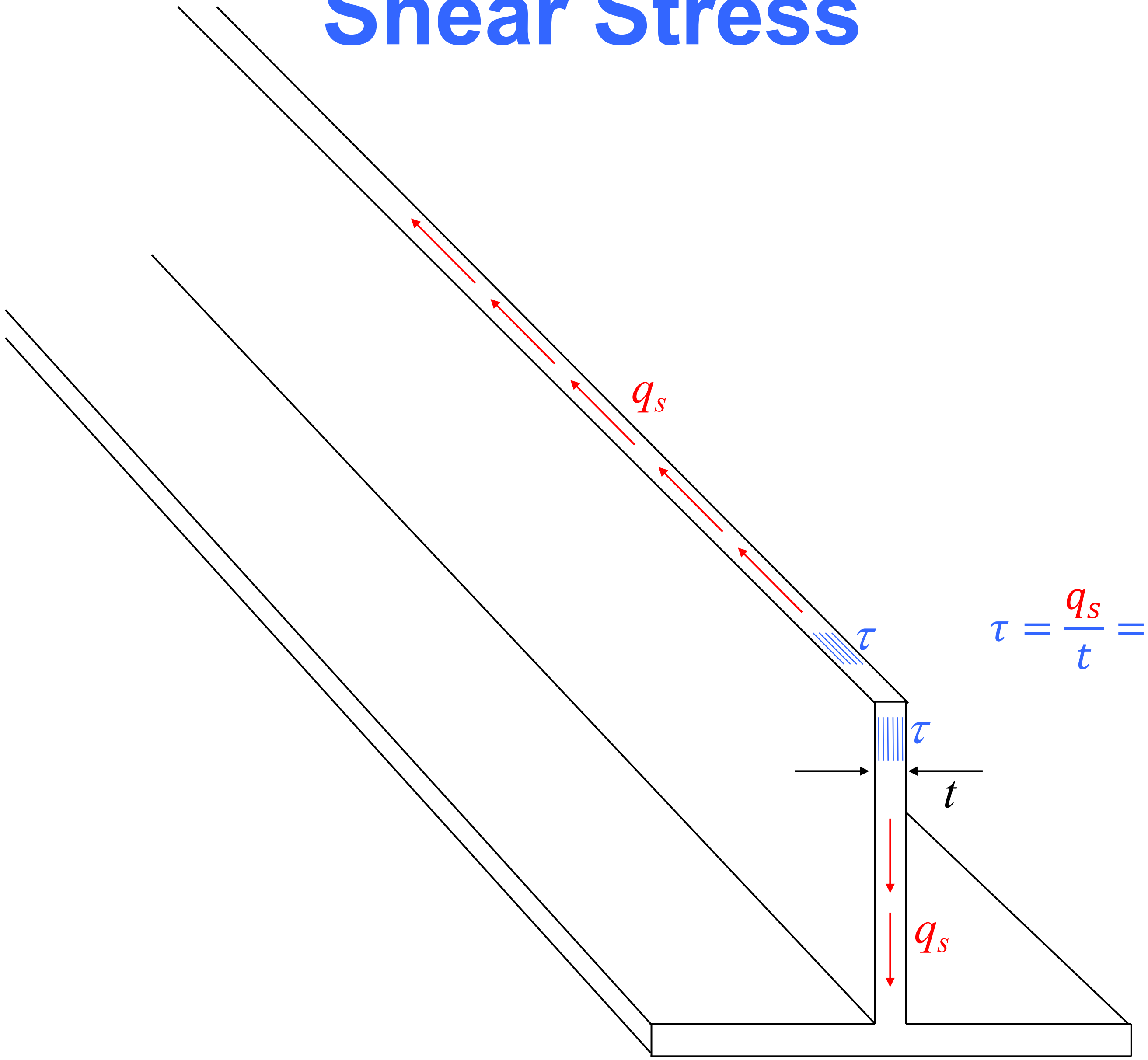




Shear Centre

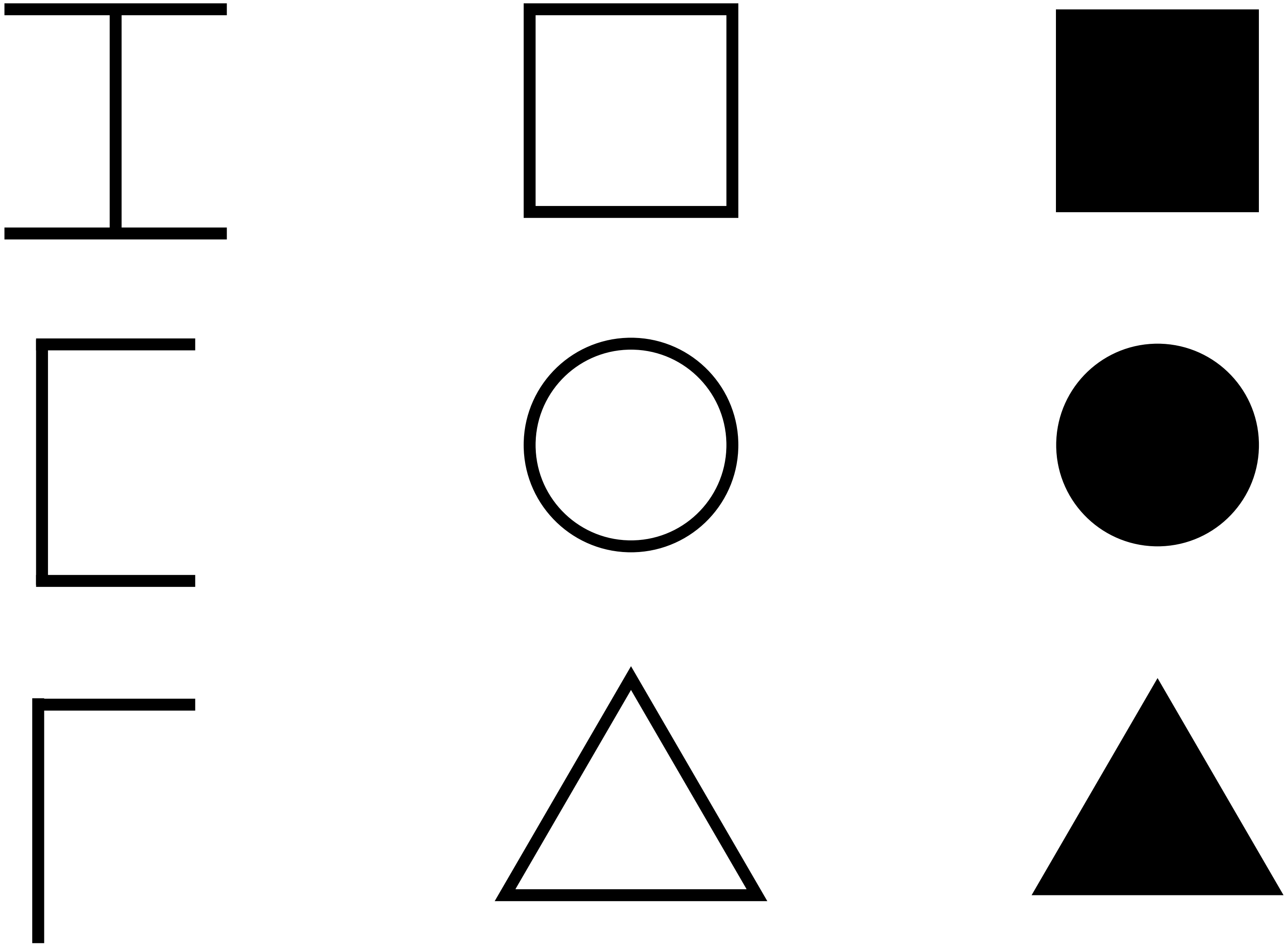


Shear Stress

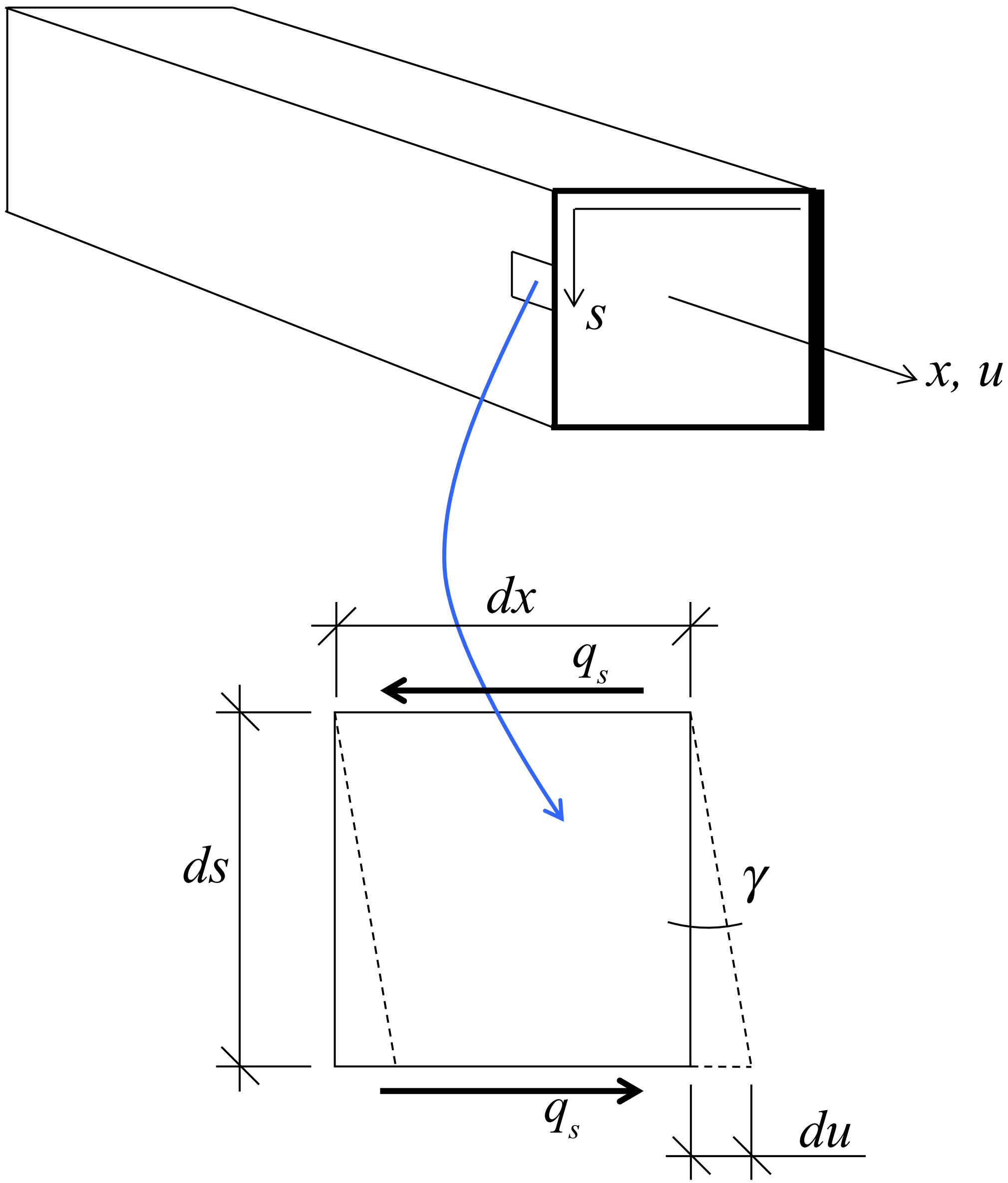


$$\tau = \frac{q_s}{t} = \frac{V \cdot Q}{I \cdot t}$$

Open, Closed, Solid



Closed Thin-walled Cross-section



$$u = \oint du = 0$$

$$\gamma = \frac{du}{ds}$$

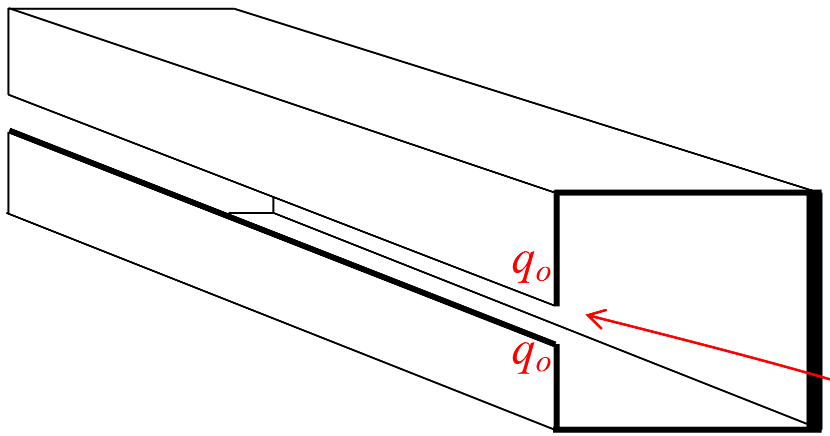
$$du = \gamma \cdot ds$$

$$\gamma = \frac{\tau}{G} = \frac{q_s}{G \cdot t}$$

$$du = \frac{q_s}{G \cdot t} \cdot ds$$

$$u = \oint \frac{q_s}{G \cdot t} ds = 0$$

Closed Thin-walled Cross-section



$$q_s(s) = q_o + \frac{V}{I} \cdot Q_{det}(s)$$

$$Q = \int_0^s z \cdot dA = \int_0^s z \cdot t \cdot ds$$

$$u = \oint \frac{q_o}{G \cdot t} ds + \oint \frac{V \cdot Q_{det}}{I \cdot G \cdot t} ds = 0$$

$$q_o = -\frac{V}{I} \cdot \frac{\oint \frac{Q_{det}}{G \cdot t} ds}{\oint \frac{1}{G \cdot t} ds} \quad Q_o$$

Evaluating Q_o

$$Q_o = -\frac{\oint \frac{Q_{det}}{G \cdot t} ds}{\oint \frac{1}{G \cdot t} ds}$$

$$q_o = -\frac{V}{I} \cdot \frac{\oint \frac{Q_{det}}{G \cdot t} ds}{\oint \frac{1}{G \cdot t} ds} \quad Q_o$$

1. Draw Q_{det} for the statically determinate cut open cross-section
2. Calculate the numerator by integration of Q_{det} divided by respective thicknesses
3. Calculate the denominator, which is straightforward because, for example, for a rectangular closed cross-section with width b, height h, and thickness t, it is simply $b/t + b/t + h/t + h/t$
4. Obtain the final Q-diagram by adding Q_o to Q_{det} from the first step, remembering the minus-sign that appears on this slide

Alternative Approach

$$Q_o = -\frac{\oint \frac{Q_{det}}{G \cdot t} ds}{\oint \frac{1}{G \cdot t} ds} = -\frac{\oint \frac{Q_{det}}{t} ds}{\oint \frac{1}{t} ds}$$

$$\oint (Q) \cdot \left(\frac{1}{Gt}\right) ds = \oint \left(\int_0^s z \cdot t \cdot d\tilde{s}\right) \cdot \left(\frac{1}{Gt}\right) ds$$

$$= \left[\left(\int_0^s z \cdot t \cdot d\tilde{s}\right) \cdot \left(\int_0^s \frac{1}{Gt} \cdot d\tilde{s}\right)\right]_o - \oint (z \cdot t) \cdot \left(\int_0^s \frac{1}{Gt} \cdot d\tilde{s}\right) ds$$

$$= -\oint (z \cdot t) \cdot \left(\int_0^s \frac{1}{Gt} \cdot d\tilde{s}\right) ds$$

$$Q_o = \frac{\oint \left(\int_0^s \frac{1}{Gt} \cdot d\tilde{s}\right) \cdot z \cdot t \cdot ds}{\oint \frac{1}{G \cdot t} ds}$$

$$Q_o = \frac{\oint \left(\int_0^s \frac{1}{t} \cdot d\tilde{s}\right) \cdot z \cdot t \cdot ds}{\oint \frac{1}{t} ds}$$

$$g(s) = \frac{\int_0^s \frac{1}{t} \cdot ds}{\oint \frac{1}{t} ds}$$

$$Q_o = \oint g(s) \cdot z \cdot t \cdot ds$$

$$Q_o = \oint g(s) \cdot z \cdot t \cdot ds + \sum \left(Q_{flange\#i} \cdot g_i\right)$$

$$q_s(s) = \frac{V}{I} \cdot \left(Q_o + Q(s)\right)$$

See examples at Terje’s toolbox (terje.civil.ubc.ca)

Known Shear Centre?

$$q_s(s) = q_o + \frac{V}{I} \cdot Q(s)$$

$$\begin{aligned} T &= \oint q_s \cdot h\,ds \\ &= \oint \left(q_o + \frac{V}{I} \cdot Q \right) \cdot h\,ds \\ &= \oint q_o \cdot h\,ds + \oint \frac{V}{I} \cdot Q \cdot h\,ds \end{aligned}$$

$$q_o = -\frac{V}{I} \cdot \frac{\oint Q \cdot h\,ds}{\oint h\,ds} = -\frac{V}{2 \cdot A_m \cdot I} \cdot \oint Q \cdot h\,ds$$

More lectures:

Terje’s Toobox:

terje.civil.ubc.ca