A short course on

Indeterminate Structures

This video:

Degree of Static Indeterminacy

Terje's Toolbox is freely available at terje.civil.ubc.ca
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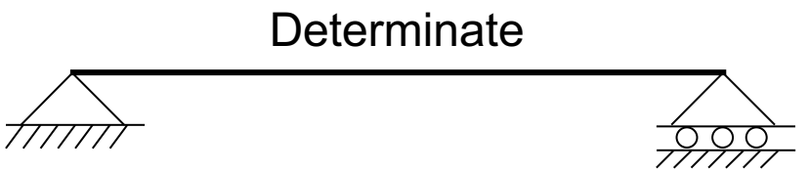
Degree of Static Indeterminacy (DSI)

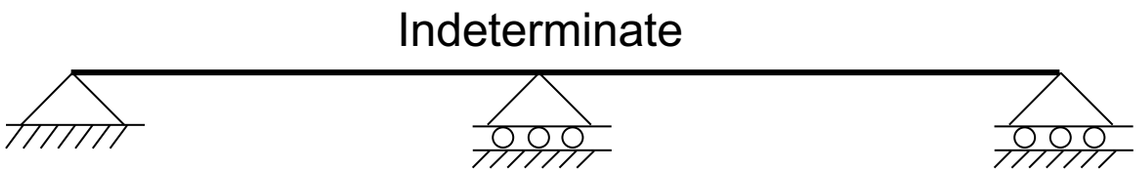
DSI = number of **unknown forces** in the structure – number of **equilibrium equations** at joints

DSI=0 means determinate

DSI=0 means equilibrium is sufficient to determine BMD, SFD, AFD

Behaviour of Determinate Structures





No need to know material behaviour to find forces

Stiffness of the structure does not influence the distribution of internal forces

Will NOT have additional forces due to support settlements or temperature changes

Do not possess redundancy

Structure will collapse if one member or one support fails

Counting

$$DSI = \left(f \cdot m + s \right) - \left(e \cdot j + h \right)$$

f	=	forces	= number of internal forces in each member
m	=	members	= number of members
s	=	support reactions	= number of support reactions, often several per support
e	=	equations	= number of equilibrium equations per joint
j	=	joints	= number of joints
h	=	hinges	= number of moment hinges or other section force releases

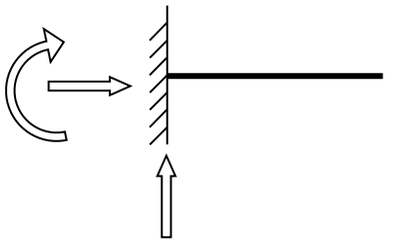
Where is the loading accounted for?

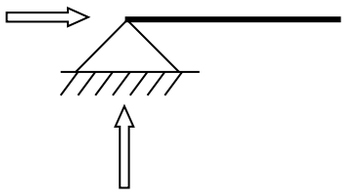
f and e

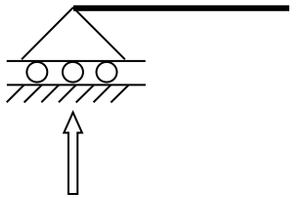
$$DSI = \left(f \cdot m + s\right) - \left(e \cdot j + h\right)$$

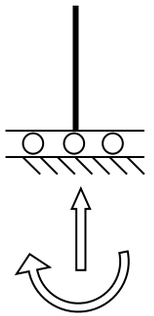
	f	e
2D truss	1	2
2D frame	3	3
3D truss	1	3
3D frame	6	6

Support Types

Fixed support:  3 reaction forces

Pinned support:  2 reaction forces

Roller support:  1 reaction force

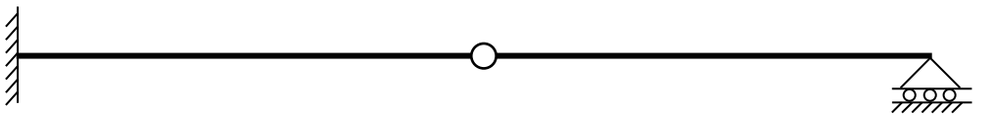
Slider support:  2 reaction forces

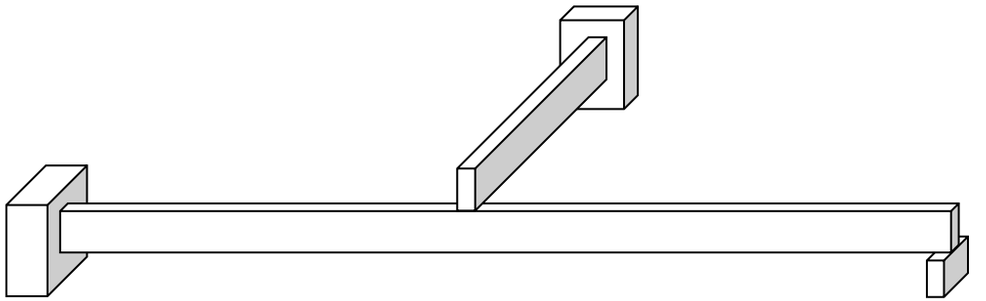
s

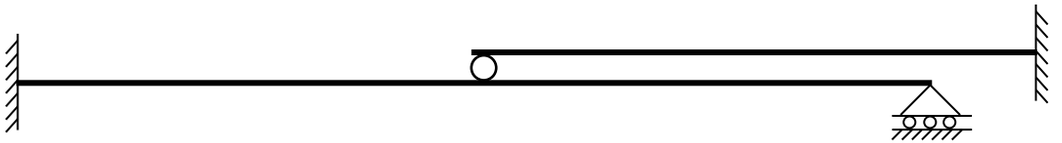
$$DSI = \left(f \cdot m + s\right) - \left(e \cdot j + h\right)$$

“Hinges”

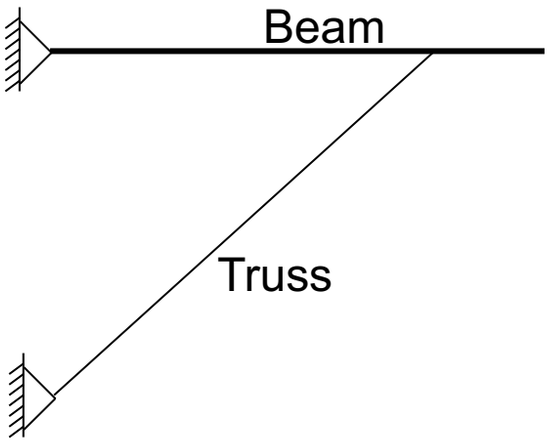
$$DSI = \left(f \cdot m + s \right) - \left(e \cdot j + h \right)$$







Counting Truss Members



Count truss member as truss member:

$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (3*2 + 1*1 + 4) - (3*3 + 1*2 + 0)$$

$$= 0 \text{ (statically determinate)}$$

Count truss member as beam/frame member:

$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (3*3 + 4) - (3*4 + 1)$$

$$= 0 \text{ (statically determinate)}$$

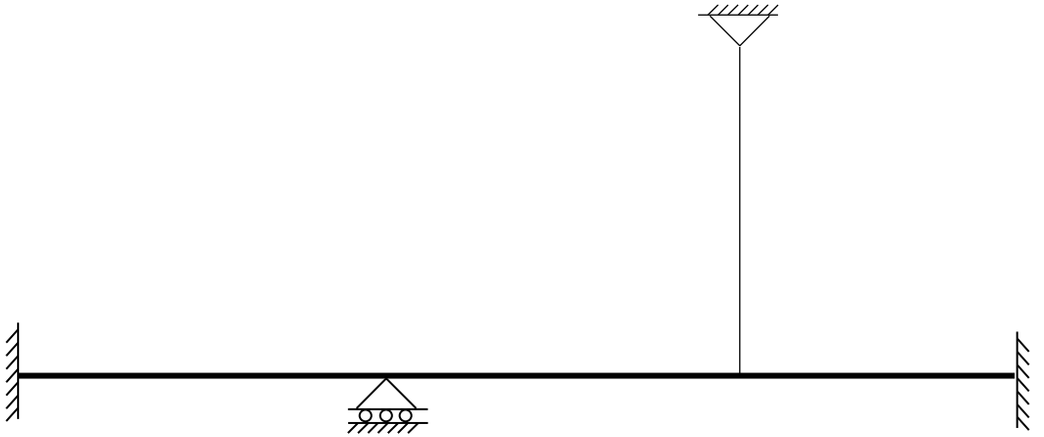
DSI vs. DKI (DOFs)

$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (3*3 + 6) - (3*4 + 0)$$

$$= 3$$

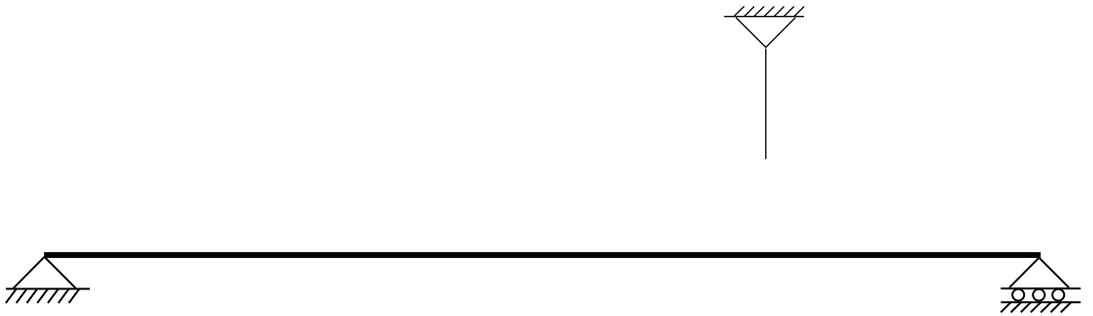
Another Way to Count



$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (3*3 + 1*1 + 9) - (3*4 + 2*1 + 0)$$

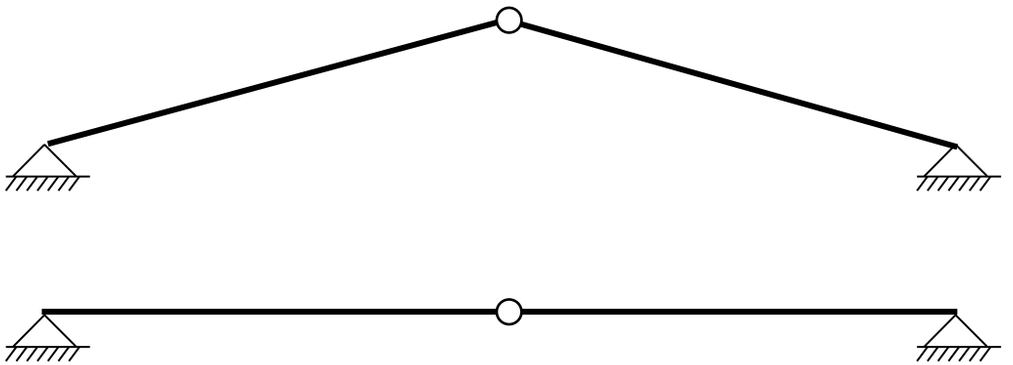
$$= 5$$



DSI vs. Stability

If DSI < 0 then the structure is unstable

DSI ≥ 0 does not guarantee stability

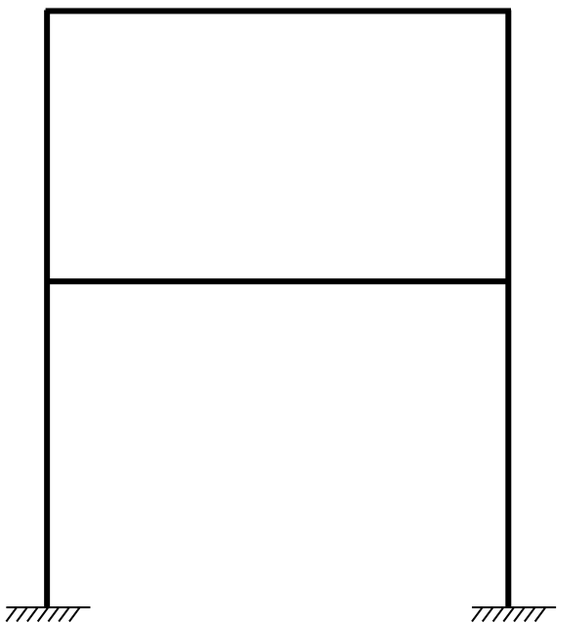


$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (3*3 + 4) - (3*3 + 1)$$

$$= 0$$

Internal & External DSI

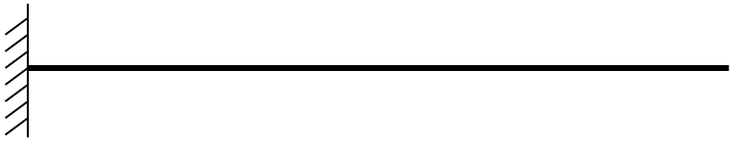


$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (3*6 + 6) - (3*6 + 0)$$

$$= 6$$

Example



$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (3*1 + 3) - (3*2 + 0)$$

$$= 0 \text{ (statically determinate)}$$

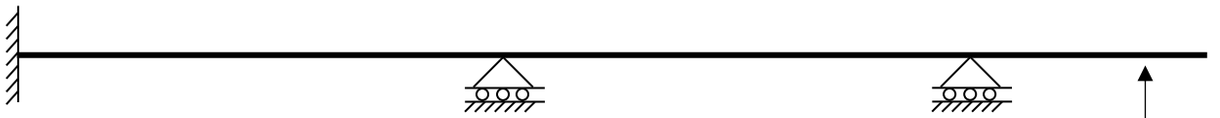
Example

$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (3*1 + 3) - (3*2 + 0)$$

$$= 0 \text{ (statically determinate)}$$

Example



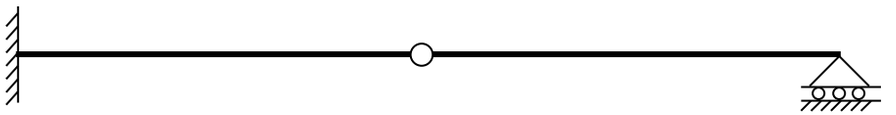
Notice that we neglect cantilevers when we count “m” and “j” because they are statically determinate entities that do not add to the DSI.

$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (3*2 + 5) - (3*3 + 0)$$

$$= 2$$

Example

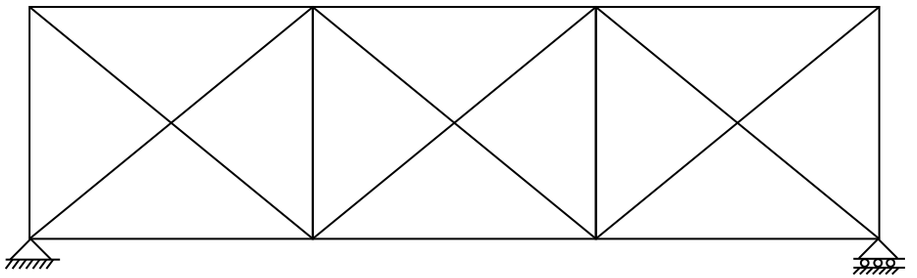


$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (3*2+4) - (3*3+1)$$

$$= 0 \text{ (statically determinate)}$$

Example

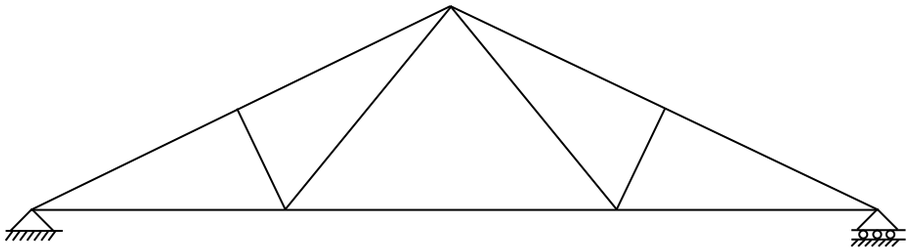


$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (1*16+3) - (2*8+0)$$

$$= 3$$

Example

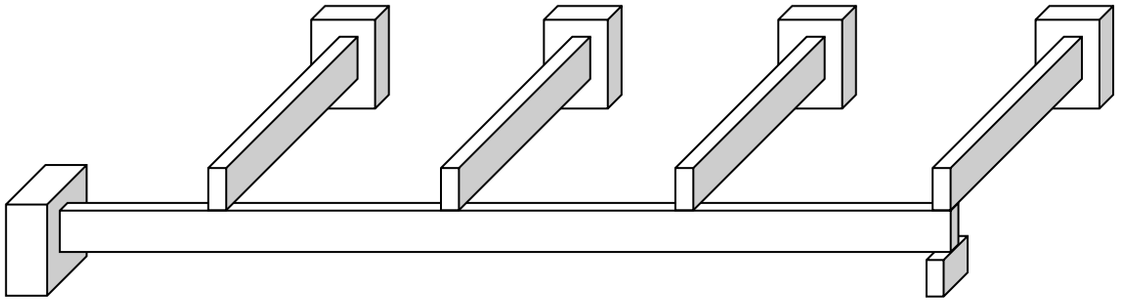


$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (1*11+3) - (2*7+0)$$

$$= 0 \text{ (statically determinate)}$$

Example



Two assumptions are made before we quantify the DSI:

- Only one force transfers between the beams
- The support on the right-hand side transfers only one force into the ground

Consider it as a **3D** structure:

$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (6*8 + 31) - (6*9 + 4*5)$$

$$= 5$$

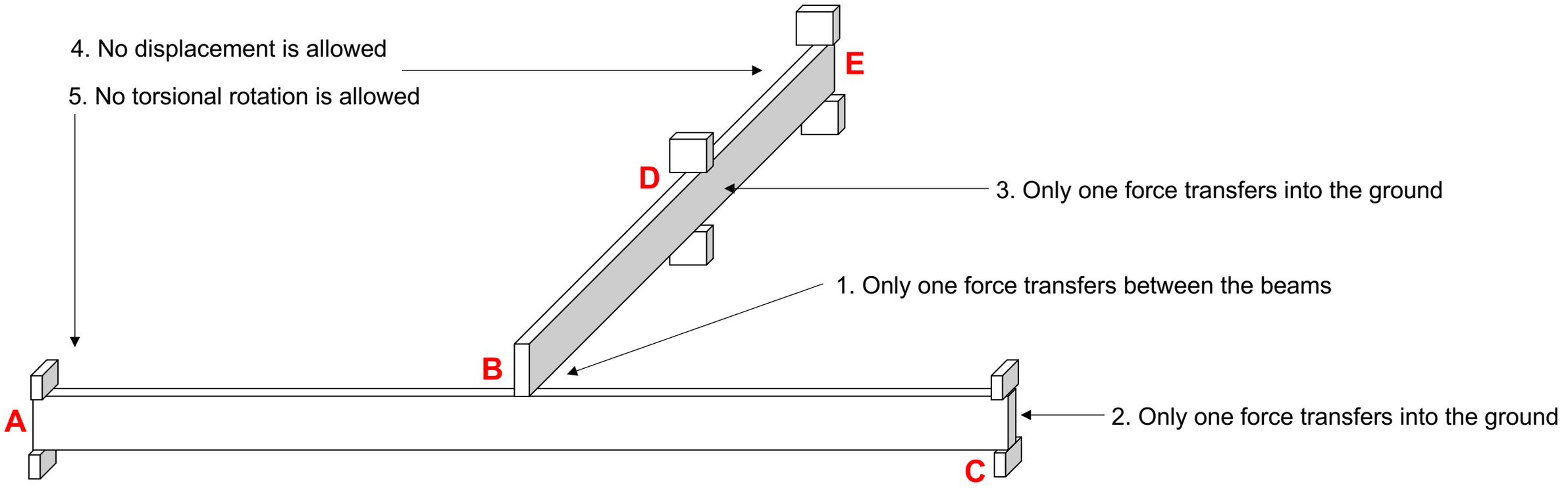
Consider it as a **2D** structure:

$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (3*8 + 16) - (3*9 + 4*2)$$

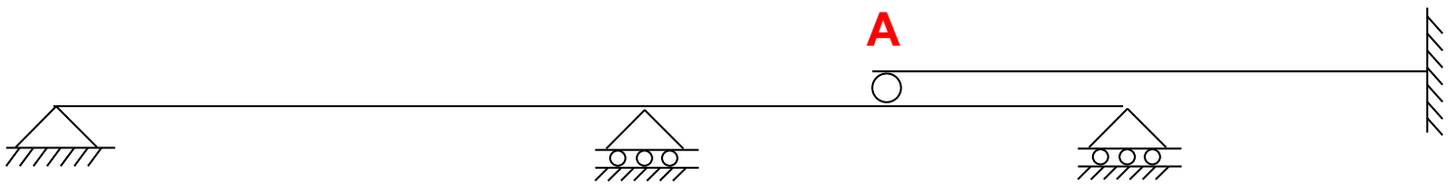
$$= 5$$

Example



$$DSI = (f \cdot m + s) - (e \cdot j + h) = (6*4 + 12) - (6*5 + 5) = 1$$

Example

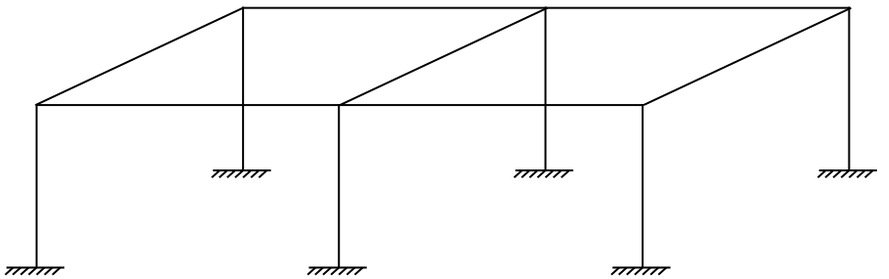


$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (3*4+7) - (3*5+2)$$

$$= 2$$

Example

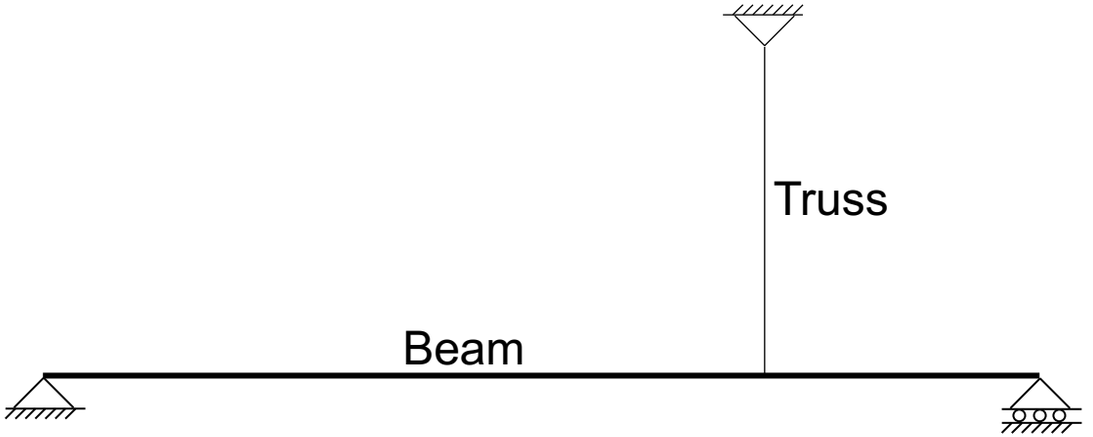


$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (6*13 + 6*6) - (6*12 + 0)$$

$$= 42$$

Example



Counting truss member as a **truss** member:

$$DSI = (f_{frame} \cdot m_{frame} + f_{truss} \cdot m_{truss} + s) - (e_{frame} \cdot j_{frame} + e_{truss} \cdot j_{truss} + h)$$

$$= (3*2+1*1+5) - (3*3+2*1+0)$$

$$= 12 - 11 = 1$$

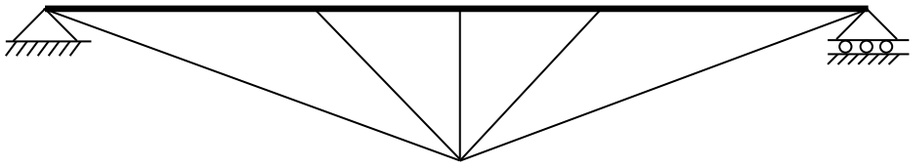
Counting truss member as a **beam/frame** member:

$$DSI = (f \cdot m + s) - (e \cdot j + h)$$

$$= (3*3+5) - (3*4+1)$$

$$= 14 - 13 = 1$$

Example



Counting truss members as a **truss** members: $DSI = (f_{frame} \cdot m_{frame} + f_{truss} \cdot m_{truss} + s) - (e_{frame} \cdot j_{frame} + e_{truss} \cdot j_{truss} + h)$

$= (3*4 + 1*5 + 3) - (3*5 + 2*1 + 0)$

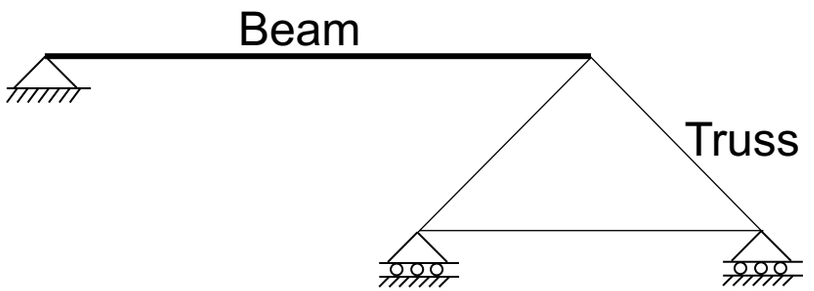
$= 3$

Counting truss members as a **beam/frame** members: $DSI = (f \cdot m + s) - (e \cdot j + h)$

$= (3*9 + 3) - (3*6 + 9)$

$= 3$

Example

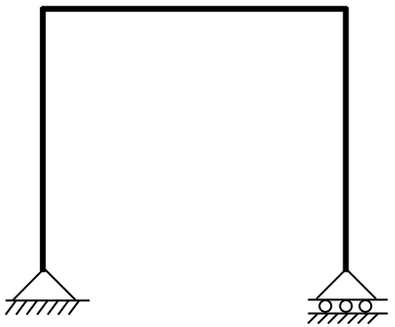


$$DSI = (f_{frame} \cdot m_{frame} + f_{truss} \cdot m_{truss} + s) - (e_{frame} \cdot j_{frame} + e_{truss} \cdot j_{truss} + h)$$

$$= (3*1 + 1*3 + 4) - (3*2 + 2*2 + 0)$$

$$= 0 \text{ (statically determinate)}$$

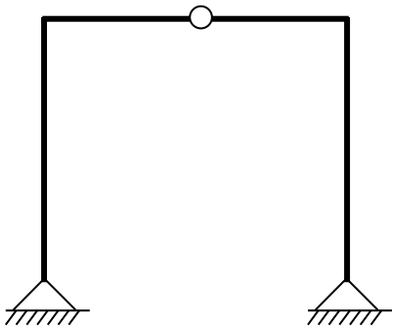
Example



$DSI = (f \cdot m + s) - (e \cdot j + h)$

$= (3*3+3) - (3*4)$

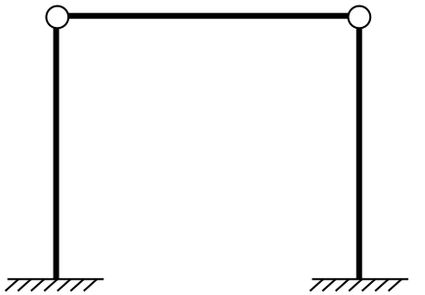
$= 0$ (statically determinate)



$DSI = (f \cdot m + s) - (e \cdot j + h)$

$= (3*4+4) - (3*5+1)$

$= 0$ (statically determinate)



$DSI = (f \cdot m + s) - (e \cdot j + h)$

$= (3*3+6) - (3*4+2)$

$= 1$

More lectures:

Terje's Toobox:

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