Forces and Resultants

Structural engineering consists of a back-and-forth between *analysis* and *design*. If the analysis reveals the design is deficient, then the engineer tweaks the structural design, for example by changing the dimensions of a structural member, followed by a new analysis. An objective in each structural analysis is to determine the deformations and internal forces caused by externally applied loads. A nominal value of each load is usually provided, together with safety coefficients, in building codes. Eurocode 1 provides load values for many jurisdictions in Europe; the National Building Code of Canada, NBCC, does the same in Canada. External loads applied to a structure, such as dead load, live load, wind load, snow load, and even the "load" from earthquake ground motion, are forces. The concept of a force is old; it is, for example, a central concept in Galileo's and Newton's work in the 1600s. Figure 1 highlights the following facts:

a) A force can cause a moment
b) A moment can be seen as a force-pair
c) A distributed load can be concentrated into a resultant force
d) Several forces acting at a point can be seen as a single resultant force
e) A single force can be decomposed into separate forces in different directions

In Figure 1a) the point load, F, at the tip of the cantilever causes a moment at the base. In Figure 1b) the distributed load, q, on the truss causes a bending moment in the middle, which is seen as a force-pair in the chords of the truss. In Figure 1c) the distributed load on the cantilever is collected into a resultant force, R, which gives the moment $M=R(L/2)=qL^2/2$ at the base. In Figure 1d) the two forces F_1 and F_2 are collected into one resultant force, R. In Figure 1e) the force, F, is decomposed into a horizontal and a vertical component.
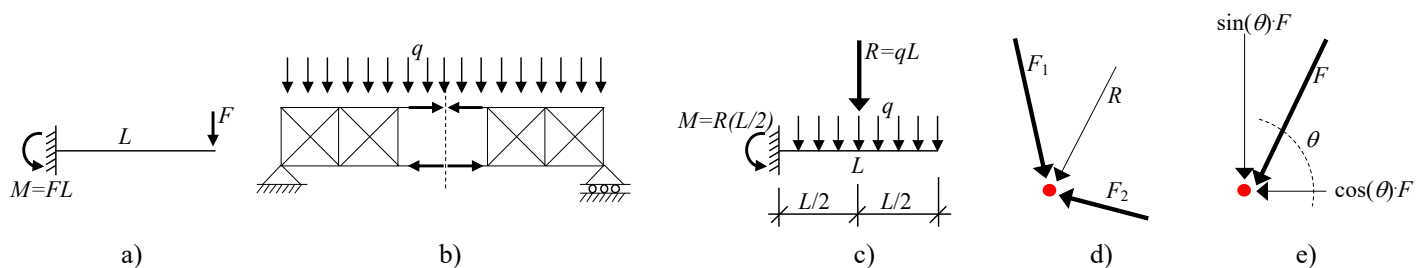

Figure 1: Loads, forces, resultants, and moments.

Finding Resultants

The concentration of the distributed load, q, in Figure 1c) into a resultant force, R, is a simple concept. Conversely, the merger of F_1 and F_2 into R in Figure 1d) requires additional attention. Two approaches are possible: trigonometry and a graphical approach. The use of trigonometry is illustrated in Figure 2. Starting from the left, Figure 2a) identifies the orientation of the two loads, relative to the axis directions. Next, in Figure 2b), those directions are employed to decompose the forces into horizontal and

vertical components, as was done in Figure 1e). Finally, in Figure 2c) those components are combined in order to determine the horizontal and vertical components of the resultant, R. The same approach is utilized if more than two forces act.
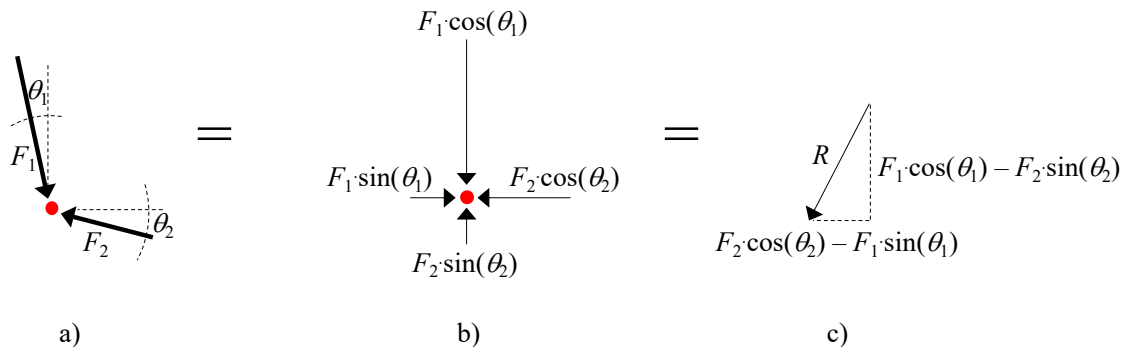

Figure 2: Finding a resultant by trigonometry.

The graphical approach for finding R is simpler, but can, depending on the case-specific geometry, lead to inaccurate results due to approximation errors. Figure 3 illustrates the approach; the forces are moved so that one follows the other. In Figure 3b) the force arrow for F_2 is moved in the plane so that it follows immediately after the end of the force arrow for F_1. The resultant force, R, is the arrow that forms a straight line between the beginning of the F_1 arrow and the end of the F_2 arrow. The same result is obtained in Figure 3c) where the F_1 arrow is moved to follow the F_2 arrow. The same approach is utilized if more than two forces act. In that case, if the end of the last arrow arrives at the beginning of the first arrow, meaning that the resultant is zero, then the forces are in equilibrium. This fact is employed in the analysis of forces in truss joints, addressed in another document.
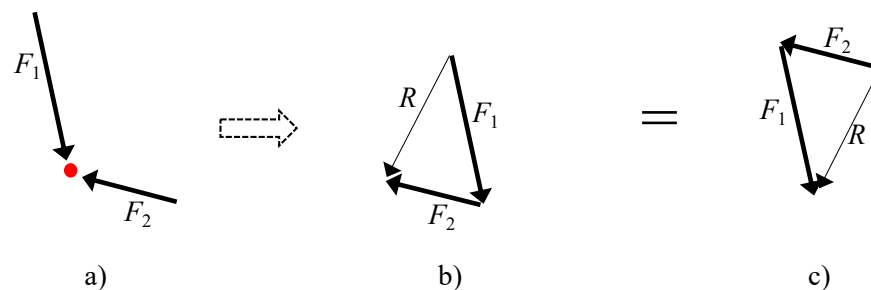

Figure 3: Finding a resultant by a graphical approach.