Complex Analysis

Complex analysis deals with complex numbers. Complex numbers have one real part and one imaginary part. One representation of a complex number is the "standard form, which reads

$$z = x + i \cdot y \tag{1}$$

where z is the complex number, x is the real part, i is the imaginary unit, and y is the imaginary part. The imaginary unit, i, is defined as

$$i = \sqrt{-1} \tag{2}$$

A graphical representation of an imaginary number is shown in Figure 1.

Consider two complex numbers, z_1 and z_2. The following equations show how arithmetic operations are carried out with them:

$$z_1 + z_2 = (x_1 + x_2) + i \cdot (\mathrm{y}_1 + y_2) \tag{3}$$

$$z_1 - z_2 = (x_1 - x_2) + i \cdot (\mathrm{y}_1 - y_2) \tag{4}$$

$$z_1 \cdot z_2 = (x_1 \cdot x_2 - y_1 \cdot y_2) + i \cdot (\mathrm{x}_1 y_2 + x_2 y_1) \tag{5}$$

$$\frac{z_1}{z_2} = \left(\frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} \right) + i \cdot \left(\frac{x_2 y_1 - x_1 y_2}{x_2^2 + y_2^2} \right) \tag{6}$$

Note that addition and subtraction of complex numbers are readily carried out by vector addition and subtraction in the complex plane in Figure 1. Each complex number has a conjugate complex number, in which the complex part has the opposite sign. I.e., $x\text{-}i\cdot y$ is the conjugate of $x\text{+}i\cdot y$. With reference to Figure 1, a complex number can be written in polar form:

$$z = x + i \cdot y = r \cdot \left(\cos(\theta) + i \cdot \sin(\theta) \right) \tag{7}$$

Euler's formula for complex numbers is

$$e^{i\omega t} = \cos(\omega t) + i \cdot \sin(\omega t) \tag{8}$$

Eq. (1) can equivalently be written as

$$z = \sqrt{x^2 + y^2} \cdot e^{i \cdot \arctan(y/x)} \tag{9}$$

If a complex number is in the denominator of a fraction then the standard form is obtained by multiplying the numerator and denominator by the complex conjugate of the denominator:

$$\frac{1}{x + i \cdot y} = \frac{1}{x + i \cdot y} \cdot \frac{(x - i \cdot y)}{(x - i \cdot y)} = \frac{x - i \cdot y}{x^2 + y^2} = \left(\frac{x}{x^2 + y^2} \right) - i \cdot \left(\frac{y}{x^2 + y^2} \right) \tag{10}$$

The real part of the following complex number is:

$$\left|\frac{a+i\cdot b}{c+i\cdot d}\right| = \sqrt{\frac{a^2+b^2}{c^2+d^2}} \tag{11}$$

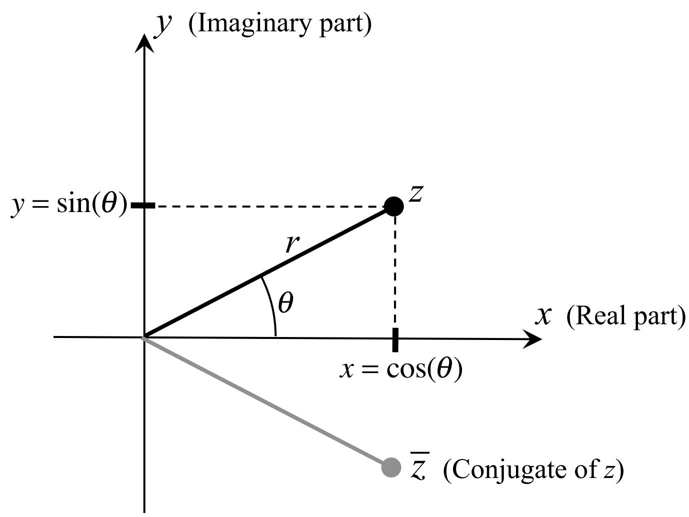

Figure 1: Graphical representation of complex number, i.e., the "complex plane."