Performance-based Design

In the early morning hours of January 17, 1994 Los Angeles awoke to violent ground shaking in what became known as the Northridge Earthquake. That shaking caused injuries and casualties, but ultimately what surprised people the most was the cost of damage to buildings and infrastructure. It is estimated that the total cost of the Northridge earthquake was, in 1994 dollars, in excess of $20 billion. That makes it one of the costliest natural disasters in U.S. history. The Northridge earthquake, and the realization that damage to buildings and civil infrastructure is so costly in a modern society, has prompted the development of performance-based engineering and performance-based design procedures.

What is Performance?

At present, performance-based design procedures are primarily employed in earthquake engineering. Some codes contain performance objectives for other loads, but earthquakes are unique, because damage is expected. Therefore, in earthquake engineering, on can state that

$$\text{Performance} = \text{Damage} \tag{1}$$

That perspective implies that the traditional design focus on stress- and stiffness-based limit-states remains, but is complemented by performance-based engineering, i.e., predictions of damage. Damage to structural and non-structural components is usually correlated with maximum displacements, measured as drift ratios. As a result, one performance-measure is

- Peak drift ratio

Certain building contents sustain damage when the maximum acceleration response is high. For that reason, one performance-measure is

- Peak acceleration

Additional structural response quantities are sometimes employed as damage predictors, i.e., performance predictors:

- Peak strain
- Plastic strain or rotation

Downstream consequences of damage are also employed as measures of performance:

- Damage indices
- Repair costs (direct cost)
- Deaths and injuries (indirect cost)
- Downtime in functionality (indirect cost)

Damage indices were popular in the 1980s and 90s while the triple-D measures (Deaths, Dollars, and Downtime) became more popular in the present century. Whether a structural response is employed, such as drift, or a downstream consequence, such as cost, the objective is usually to predict its probability distribution. Figure 1 shows a sketch of the typical result, i.e., the probability that the cost, *C*, exceeds the threshold *c*, and the probability that the drift, *D*, exceeds the threshold *d*. A rational decision criterion is to minimize the expected, i.e., mean cost or drift. As shown in the document on Random Variables on this website, that mean value is the area underneath the curves in Figure 1.

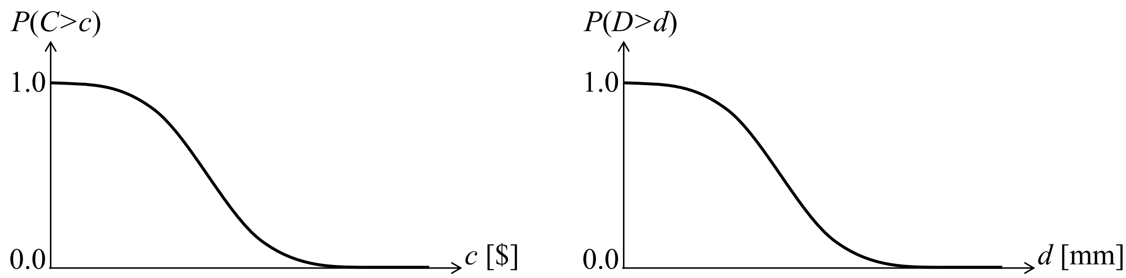

Figure 1: Probability distributions for cost and displacement response.

An extended view of performance-based engineering is to consider the impacts on the city or community that sustains damage to its buildings and infrastructure. The question is often how quickly the community can bounce back after the earthquake. For that reason, an performance-measure is

- Resilience

Resilience has to do with recovery after a shock. The term was used in the 1970s in the context of ecological systems subjected to adversity (Holling 1973). Figure 2 shows a "resilience curve," i.e., the functionality level of, say, a community after a shock, such as an earthquake. Curves like that are intuitive, but often it is necessary to dig deeper to understand the resilience of a system. Questions that quickly emerge include: What is the measure of functionality? What are the models that predict the drop in functionality? What are the models that predict the predict the recovery of the functionality?

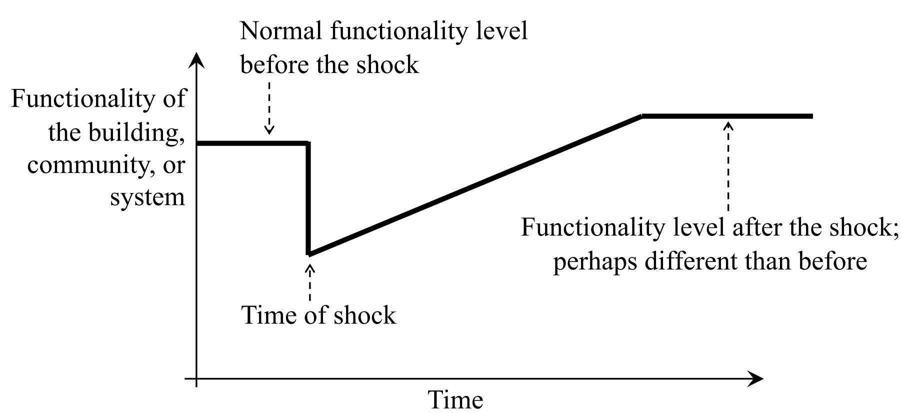

Figure 2: A resilience curve.

In the context of seismic resilience of communities an Earthquake Spectra paper from 2003 has been influential (Bruneau et al. 2003). That paper puts forward the following four Rs, which are also included in Figure 3:

- Robustness: Ability to withstand the shock
- Redundancy: Ability of secondary elements to carry the effects of the shock
- Resourcefulness: Availability of resources to recover after the shock
- Rapidity: Timeliness in making those recourses available after the shock

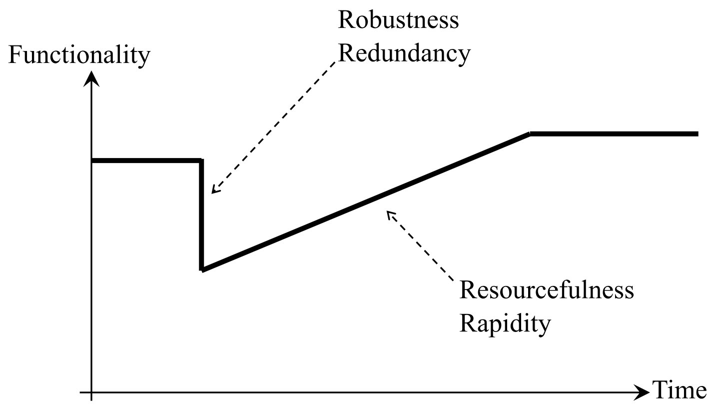

Figure 3: Definitions related to resilience.

Resilience also has to do with systems and communities. That means that the performance of buildings is one part of a bigger picture. The functionality of critical societal functions and critical infrastructure is a paramount concern in resilience studies. The following items are often included as societal critical functions: 1) Habitat; 2) Power; 3) Water; 4) Fuel; 5) Transportation; 6) Telecommunications; and 7) Waste removal. Structural engineers influence several of those items. The design of buildings, i.e., habitat to withstand earthquakes is one example. The design of bridges in a transportation network is another example. Structures that are part of power grids and telecommunication grids are also within the scope of structural engineers. Several documents on this website are relevant for the reparability of such structures.

PEER Triple Integral

In terms of performance-based engineering methodologies, the "PEER equation" has been influential. It was the "framing equation" for the research conducted in the Pacific Earthquake Engineering Research (PEER) Center (Cornell and Krawinkler 2000; Moehle and Deierlein 2004). It defines performance by means of a decision variable, *DV*, such as the monetary cost of potential repairs. The other variables are *DM*=damage measure, *EDP*=engineering demand parameter, such as drift, and *IM*=ground motion intensity measure. The triple integral appears due to three applications of the theorem of total probability:

$$G(dv) = \int_0^\infty \int_0^\infty \int_0^\infty G(dv \mid dm) \cdot \left| \frac{\mathrm{d}G(dm \mid edp)}{\cancel{\mathrm{d}dm}} \right| \cdot \left| \frac{\mathrm{d}G(edp \mid im)}{\cancel{\mathrm{d}edp}} \right| \cdot \left| \frac{\mathrm{d}G(im)}{\cancel{\mathrm{d}im}} \right| \cancel{\mathrm{d}dm} \cdot \cancel{\mathrm{d}edp} \cdot \cancel{\mathrm{d}im} \quad (2)$$

where *G*=complementary cumulative distribution function. If *G*(*im*) is replaced by the hazard function $\lambda(im)$, which gives the annual rate of exceedance of the intensity *im*, then Eq. (2) predicts the annual exceedance rate of *dv*, i.e., $\lambda\,(dv)$ instead of *G*(*dv*).

Priestley

An early example of using drift as a proxy for performance is the FEMA 273 guidelines (Federal Emergency Management Agency 1997). Professor M.J. Nigel Priestley in New Zealand was an early proponent of this approach (Priestley 1993) and he developed neat step-by-step design procedures with target drift limits (Priestley et al. 2007). One application is wood-frame buildings (Filiatrault and Folz 2002). The key steps are:

1. Select a target maximum drift; i.e., select the target performance
2. Determine the energy dissipated at that target drift; the dissipation is measured by the equivalent viscous damping ratio, determined from experiments on various structural systems
3. Determine the equivalent elastic period of vibration of the structure
4. Determine the required lateral stiffness of the structure
5. Once the structural system is designed to match the required stiffness the base shear is calculated for a capacity check

Cornell

Professor C. Alin Cornell at the University of Stanford was an important contributor to the performance-based methodologies that in a sense grew out of the Northridge earthquake. The PEER equation presented earlier is one example. Another example is the analytical expression for the probability distribution for drift, developed by Cornell et al. (2002). Here using $G(x)$ as notation for the complementary CDF of a random variable X, the derivations are:

1. Express the hazard in terms of spectral acceleration, S_a, as $G(S_a) = k_o \cdot S_a^{-k}$
2. Express the relationship between S_a and drift, D, by a lognormal CDF with median $\hat{D} = a \cdot S_a^b$ and dispersion $\beta_{D|S_a}$: $G(d \mid S_a) = 1 - \Phi\left(\frac{1}{\beta_{D|S_a}} \ln\left(\frac{d}{a \cdot S_a^b} \right) \right)$
3. Apply the theorem of total probability to obtain the unconditional probability distribution for drift: $G(d) = \int_0^\infty G(d \mid S_a) \cdot \left| \frac{dG(S_a)}{dS_a} \right| dS_a = k_o \cdot \left(\frac{d}{a} \right)^{-\frac{k}{b}} \cdot e^{\frac{k^2}{2b^2} \cdot \beta_{D|S_a}^2}$
4. In this "SAC procedure" developed by Cornell, the probability that the drift exceeds the drift capacity, C, i.e., the failure event, F, is expressed by a lognormal CDF with median $\hat{C}$ and dispersion β_C: $P(F \mid d) = \Phi\left(\frac{1}{\beta_C} \ln\left(\frac{d}{\hat{C}} \right) \right)$
5. Applying the theorem of total probability to yields the unconditional probability of failure: $P(F) = \int_0^\infty P(F \mid d) \cdot \left| \frac{dG(d)}{\mathrm{d}d} \right| \mathrm{d}d = k_o \cdot \left(\frac{\hat{C}}{a} \right)^{-\frac{k}{b}} \cdot e^{\frac{k^2}{2b^2} \cdot \left(\beta_{D|S_a}^2 + \beta_C^2 \right)}$

Moehle

Eq. (2) was intended as a framing equation to coordinate the work in the PEER Center and is not particularly appealing for actually obtaining "loss curves," i.e., probability distributions for cost. Conditional probability distributions do not necessarily represent a good modelling approach, and may inhibit future model improvement efforts. PEER Director Jack Moehle, Professor at the University of California at Berkeley took the initiative to extend the PEER methodology for performance-based engineering to include detailed analysis of specific buildings (Yang et al. 2009). That approach, which became known as the ATC-58 approach and now embedded in the FEMA P-58 guidelines, implies that finite element analysis is part of the analysis. Notice that apart from the naming of variables there is little resemblance to Eq. (2). ATC-58 procedure is:

1. Select a few hazard levels, e.g., 2% in 50 years, 5% in 50 years and 10% in 50 years
2. For each hazard level, collect a suite of ground motions, perhaps by scaling recorded ground motions
3. For every hazard level:
    a. For every ground motion, analyze the building using a finite element program, such as OpenSees, which was developed at the PEER Center (McKenna et al. 2009)
    b. After recording the "EDPs," e.g., the inter-storey drifts and acceleration responses for every ground motion, a matrix of results is created; each row represents a ground motion number and each column, $\mathbf{X}_i$, contain realizations of an EDP
    c. Each column, $\mathbf{X}_i$, of the aforementioned matrix are seen as realizations of the random variable X_i, i.e., EDP number i, and the random variables X_i are assumed to have the joint lognormal distribution
    d. Take the natural logarithm of all entries in the aforementioned matrix and let the new values in the columns be denoted by $\mathbf{Y}_i$, which are each seen as realizations of a normally distributed random variable
    e. Calculate the second-moment information for the columns $\mathbf{Y}_i$, i.e., the mean vector, $\mathbf{M}_Y$, and the covariance matrix, $\mathbf{\Sigma}_{YY}$; the joint lognormal distribution is now essentially established
    f. Use a random number generator to generate realizations of standard normal random variables $\mathbf{U}$
    g. Use the second-moment probability transformation explained elsewhere on this website to obtain additional but synthetic realizations of the normal variables $\mathbf{Y}=\mathbf{M}_Y+\mathbf{D}_Y\mathbf{L}\mathbf{U}$, where $\mathbf{L}$ is the lower-triangular Cholesky decomposition of the correlation matrix embedded in $\mathbf{\Sigma}_{YY}$
    h. Take the exponent of the $\mathbf{Y}$-values to obtain synthetic realizations of the $\mathbf{X}$-values
    i. For each of the many new $\mathbf{X}$-values, i.e., EDP-values now available:
        i. Enter the value into a set of fragility curves to determine the probability of different damage states for that EDP value; i.e., determine $P(DS_1)$, $P(DS_2)$, $P(DS_3)$, and $P(DS_4)$

ii. Generate the outcome of a random variable uniformly distributed between 0 and 1 and let that realization determine one damage state, using the probabilities from the previous step
iii. Now having the damage state, use unit repair cost functions to determine the cost of the repairs for this EDP-value

The ATC-58 procedure is implemented in the computer program PACT, and represents one of the most detailed performance-based engineering analysis procedures utilized in practice. The idea of increasing the flexibility of the modelling compared with Eq. (2) was also the motivation behind the work of Mahsuli and myself at UBC in Vancouver. Here an object-oriented computer program called Rt was developed in C++ to allow "plug and play" with models in the same manner that OpenSees facilitates plug and play with methods. Specifically, in OpenSees it is straightforward to replace an equation solver by another without changing anything in the code. The framework and models implemented in Rt, together with several applications to buildings and regions near Vancouver, were published a while ago (Mahsuli and Haukaas 2013a; b; c).

Damage Indices

Employed primarily in the 1990s, a technique to quantify damage to a structure or an element is to use damage indices. Perhaps the most famous is the "Park and Ang" index (Park and Ang 1985), which is defined as

$$D = \frac{\delta_M}{\delta_u} + \frac{\beta}{Q_y \delta_u} \int dE \tag{3}$$

where δ_M=maximum deformation, δ_u=ultimate deformation in experiments with monotonic loading, β=model parameter, Q_y=yield strength, and E=absorbed hysteretic energy. Eq. (3) is formulated so that D>1 signifies "complete collapse or total damage" (Park and Ang 1985). The second term on the right-hand side of Eq. (3) accounts for cyclic loading but Park and Ang suggest that it may be insufficient to simply add up the hysteretic energy to predict damage, as that term does. To address that, they propose an extended version of Eq. (3), in which the displacement affects the damage accumulated by that term:

$$D = \frac{\delta_M}{\delta_u} + \beta \cdot \int \left(\frac{\delta}{\delta_u} \right)^{\alpha} \cdot \frac{1}{E_c(\delta)} dE \tag{4}$$

where α=model parameter and $E_c(\delta)$=hysteretic energy in a cycle that is associated with the displacement δ. A substantial portion of the paper by Park and Ang is devoted to the determination of the model parameters δ_u, Q_y, and β in Eq. (3). Williams and Sexsmith (1995) provide a didactic review of several damage indices applicable to reinforced concrete, categorizing the indices in the following manner:

- Local element/connection indices
    - Non-cumulative indices
    - Cumulative indices
        - Deformation-based

- Energy-based
  - Combined non-cumulative and cumulative indices, such as Eq. (3) above
- Global structure-level indices
  - Weighted average indices

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