Uncertainty

The concept of uncertainty is philosophically interesting and practically consequential. All humans face uncertainty but we react to it in different ways. Uncertainty permeates most human endeavours and we often find ourselves making decisions under conditions of uncertainty. In fact, from a philosophical viewpoint, this is the essence of engineering. It has been argued that humans strive to increase predictability and reduce uncertainty. Studies show that people faced with different decision alternatives prefer the option with least uncertainty. The notion that uncertainty as an undesirable attribute is also reflected in humans who cherish past eras because of the comforting certainty about what happened the next day. The present day is different; we feel uncertain about whether the stock market will crash tomorrow and whether there will be a big earthquake next year. On the other hand, it is interesting to note that while uncertainty is often viewed negatively and we seek to reduce it, many people express personal reluctance about getting to know with certainty what will occur in their lives in the future.

Categorization of uncertainty is somewhat controversial. The labels "aleatory" and "epistemic" often appear, but their significance is debated (Der Kiureghian and Ditlevsen 2009). These notes adopt the pragmatic approach that categorization is useful when it has a practical implication. In this case, the implication is that uncertainty is either reducible or not. In other words, the labels are embraced for the purpose of distinguishing between the uncertainty that can be reduced by human intervention and that which cannot. This renders possible engineering decisions to allocate resources to reduce the uncertainty before making design decisions. To this end, the two categories of uncertainty are described in the next two subsections.

The Nature of Uncertainty

Aleatory and Epistemic

Aleatory uncertainty is irreducible. The only way to avoid this uncertainty is to change the nature of the phenomenon under consideration. Philosophically, the presence of this type of uncertainty is debated. One may question whether anything is inherently uncertain. In other words, it may be argued that it our models that poorly reflect reality. The discussion of Laplace's demon and Heisberg's uncertainty principle provides the extreme contrasts for this discussion. However, regardless of philosophy, in engineering practice the notion that all uncertainty could be removed belongs to utopia. In short, irreducible uncertainty is unavoidable in engineering. Conversely, epistemic uncertainty is reducible. This type of uncertainty is reduced by, e.g., gathering more data or by improving the model. In other words, epistemic uncertainty is related to our state of knowledge, which can often be improved. Notice also that knowledge can be subjective; what is known to one analyst may be unknown to another.

Transition from Aleatory to Epistemic

While it may sound strange at first that aleatory uncertainty can transition to become epistemic, this is a trivial matter from a practically perspective. It is possible to come up with many examples where irreducible uncertainty becomes reducible, fore example because of the passing of time. As an illustration, consider the strength of concrete, measured by the reference cylinder strength. In the design of a future building it is impossible to eliminate all uncertainty in the concrete strength; many concrete cylinder specimens from the manufacturer can be tested but aleatory uncertainty will remain. Now suppose the engineer carries out additional work on the building after the building is completed. The engineer remains uncertain about the concrete strength. However, the uncertainty has transitioned from aleatory to epistemic because the engineer can eliminate the uncertainty by core drilling tests. The client cannot afford such tests, and the engineer remains uncertain about the concrete strength, but it is now epistemic in nature.

Laplace’s Demon and Heisenberg’s Uncertainty Principle

In 1814 Laplace published the influential idea that if we knew the precise initial condition of every particle in the universe at one time, then we could deterministically predict everything in the future. In other words, aleatory uncertainty does not exist; all uncertainty could be reduced by human efforts. The devise by which this could be achieved is called Laplace’s demon. The easy route to explain why this is not achievable is that it would take far too much effort. However, in 1927 Werner Heisenberg formulated the uncertainty principle, which states that the location and velocity of an electron cannot both be determined, not because of limitations in measuring devices but as a physical property. In other words, nature itself is associated with aleatory uncertainty. This notion has been debated by the most prominent of researchers. In response to the uncertainty principle Albert Einstein suggested that God does not play dice, which prompted Niels Bohr to instruct Einstein not to tell God what to do. The uncertainty principle was also rebutted by Erwin Schrödinger, who devised an imaginary experiment that became known as Schrödinger’s cat. Nevertheless, the uncertainty principle, which perhaps should be renamed to the principle of indeterminacy, still stands as a cornerstone in quantum physics. And regardless of its interpretation there, in the realm of engineering it serves as a reminder that Laplace’s demon will always remain Utopian; aleatory uncertainty will always exist in engineering applications.

Sources of Uncertainty

When assessing if a design is safe enough it is important to be aware of sources of uncertainty. It is prudent to continually ask if the list of sources is exhaustive, and how each source of uncertainty is modelled. The list will be problem-specific but some common sources of uncertainty are listed below.

Inherent Uncertainty

Most predictions of physical phenomena are associated with inherent uncertainty. For example, it is inconceivable that models will become available to predict concrete strength and earthquake magnitude with certainty. Inherent uncertainty is aleatory.

Statistical Uncertainty

Lack of data yields uncertainty that is reduced by gathering of more data. Statistical uncertainty is epistemic.

Model Uncertainty

A model is imperfect either because it has the wrong form and/or because influential parameters are missing. Some argue that the phrase model error should be employed and that the error is not uncertain. In these notes, however, model error is considered uncertain unless it is known. Model error is considered epistemic because it can be reduced by modeling efforts, except some of the uncertainty in potential missing parameters, which may be aleatory.

Measurement Uncertainty

The deviation between real values and those obtained by imperfect measurement devices is referred to as measurement uncertainty. When present, this source of uncertainty is considered aleatory because it cannot be reduced unless the entire measurement approach is replaced.

Human Error

Human error is the source of a significant portion of structural failures. However, accounting for it by probabilistic modelling is as difficult as it is important. This source of uncertainty is to some extent reducible by implementation of quality assurance regimes, such as peer inspection of calculations. However, part of this uncertainty remains aleatory, which must not be forgotten.

Description of Uncertainty

The primary means of expressing uncertainty is probability. As described elsewhere in these notes, a probability is a number between zero and one that conveys either a recurrence frequency or a degree-of-belief in the occurrence of an event. For example, we may be uncertain about the magnitude of an impending earthquake. A magnitude in excess of 8 is an example of an event. In other words, probabilities are used to quantify our uncertainty about the occurrence of events. Other approaches, such as evidence theory and fuzzy sets, have been put forward. However, when the dust settles the use of probabilities is both appealing and sufficient. On the other hand, proxy measures of probability are useful. Likelihood and entropy are examples. For example, entropy expresses the difficulty associated with making a prediction. High entropy implies low predictability, and vice versa. From this viewpoint, engineering modelling can be viewed as an effort to minimize entropy.

The distinction between aleatory and epistemic uncertainty sometimes leads researchers to stray from the use of probabilities to describe epistemic uncertainty. However, this leads to potential inconsistencies and these notes advocate the use of probabilistic concepts to characterize both aleatory and epistemic uncertainty. Using the example of concrete strength, which transitions from aleatory to epistemic once the building is built, should the engineer characterize this uncertainty differently than when he first designed the building? These notes say no; the uncertainty is the same, although it now can be reduced with some effort.

Definitions of Risk

In colloquial language the word risk is used to identify circumstances that entail some kind of danger. Perhaps surprisingly, the definition of risk is still debated in the academic community (Aven 2012; Bernstein 1998). However, some fundamental concepts related to risk in engineering exist, as described in the following.

Discrete Consequences

In the case where the risk associated with a particular "failure event" is contemplated, the following definition is adopted:

$$\text{Risk} = p_f \cdot c_f \tag{1}$$

where p_f is the probability of failure and c_f is the cost of failure. This definition of risk as "probability times consequence" means that the following two scenarios are associated with the same risk:

- Low probability of occurrence and high cost
- High probability of occurrence and low cost

Another implication of this risk definition is that risk can be interpreted as expected cost. In other words, if many "trials" take place, each with the same failure probability and failure cost, then over time the appropriate risk-based decision criterion is to try to minimize the product in Eq. (1). Other documents in these notes expand the definition of risk-based decision criteria.

Continuous Consequences

In situations where no tangible failure event is defined, the definition of risk gets more complicated. Consider the important example of an uncertain monetary loss, which is characterized by a random variable. Seismic loss due to repair and downtime is one example; financial loss of invested assets is another. In this case it is debatable how risk should be quantified. The financial sector has seen a shift from risk measures like "value at risk," i.e., quantiles, to "coherent" risk measures like conditional value at risk (Artzner et al. 1999; Rockafellar 2007). The transfer of these risk measures to other engineering applications is still debated. In this context it is important to note that the probability distribution for the total cost is the all-encompassing measure of risk, and that all scalar risk measures are extracted from it. The probability distribution is often presented as a complementary cumulative distribution function and called "exceedance probability" curve, or EP curve.

Reliability, Vulnerability, Resilience, and Robustness

While risk is a central concept when dealing with probabilities and consequences in engineering applications, other terms often appear. In these notes, "reliability" is an important term that is addressed elsewhere, with the following fundamental definition:

- Reliability equals the failure probability subtracted from unity, i.e., $1-p_f$

As a result, risk is related to reliability analysis, in the sense that reliability analysis provides the event probability in Eq. (1). Additional terms, which serve to broaden the description of the consequences of failure, are also employed:

- Vulnerability is a measure of the direct consequences of failure
- Robustness is a measure of the indirect consequences failure
- Resilience is a measure of the cost and time of recovery

References

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