## Discounting

Discounting is an important concept when making decisions that involve comparisons or summations of present and future costs. For several reasons, humans prefer costs to materialize in the future instead of now. Equivalently, we want benefits now instead of later. To understand discounting, it is useful to know that it is the inverse of compounding. In compounding we calculate the future value of present money; in discounting we calculate the present value of future money. Regardless of application, the factor between present and future money can be very large. Albert Einstein suggested humorously suggested that compounding is the eighth wonder of the world and Warren Buffet joked that Queen Isabel should have rather placed in the bank at $4 \%$ interest the $\$ 30,000$ she gave to Columbus to discover America because it would be worth $\$ 26$ trillion today. This is verified by studying the discount/compound factor, here denoted $\delta$, which says

$$
\begin{equation*}
\text { Future compunded value }=\delta \cdot \text { Present value } \tag{1}
\end{equation*}
$$

and conversely

$$
\begin{equation*}
\text { Present discounted value }=\frac{\text { Future value }}{\delta} \tag{2}
\end{equation*}
$$

## Discrete vs. Continuous

The expression for the factor $\delta$ depends on whether we use discrete or continuous compounding/discounting. In practical terms, discrete compounding means that we get the accumulated interest at regular intervals, e.g., once a year so that interest on that extra money only starts accumulating after that. Then the formula is

$$
\begin{align*}
\delta & =(1+r) \cdot(1+r) \cdots(1+r) \\
& =(1+r)^{n} \tag{3}
\end{align*}
$$

where $r=$ interest rate per time interval, e.g., annual interest rate, and $n=$ number of intervals, e.g., number of years. The continuous version is

$$
\begin{equation*}
\delta=e^{r-t} \tag{4}
\end{equation*}
$$

where $t=t i m e ~ p e r i o d ~ m e a s u r e d ~ i n ~ t h e ~ s a m e ~ u n i t ~ o f ~ t i m e ~ t h a t ~ t h e ~ i n t e r e s t ~ r a t e ~ i s ~ s p e c i f i e d ~$ in. The difference between the discrete and continuous version is small but continuous compounding is better: with $3 \%$ interest for 50 years you have $\delta=e^{(0.03)(50)}=4.48$ times the original money and "only" $\delta=(1+0.03)^{50}=4.38$ with discrete compounding.

## Selecting Lifecycle Duration

Table 1 shows the value of $\delta$ for combinations of rate and period using the continuous approach. From that table one can state facts like an annual price growth of $7 \%$ for real estate leading to a doubling of prices in 10 years. Similarly, using a discount rate near 5\%
implies that a cost occurring in 50 years does not even mean a tenth of what the same cost would today. Such considerations can enter into the selection of lifecycle duration for the analysis of a building. Suppose we are confident the building will be around for 50 years, and that we wish to apply a $5 \%$ discount rate to all costs. Costs accruing beyond the $50-$ year time horizon will enter with less than a twelfth of its future value in our calculations. That means it matters little if we select 50 years or 60 years as lifecycle duration.

Table 1: Compound/discount factors.

|  | $1 \%$ | $2 \%$ | $3 \%$ | $4 \%$ | $5 \%$ | $6 \%$ | $7 \%$ | $8 \%$ | $9 \%$ | $10 \%$ | $15 \%$ | $20 \%$ | $25 \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 year | 1.0 | 1.0 | 1.0 | 1.0 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.2 | 1.2 | 1.3 |
| 2 years | 1.0 | 1.0 | 1.1 | 1.1 | 1.1 | 1.1 | 1.2 | 1.2 | 1.2 | 1.2 | 1.3 | 1.5 | 1.6 |
| 3 years | 1.0 | 1.1 | 1.1 | 1.1 | 1.2 | 1.2 | 1.2 | 1.3 | 1.3 | 1.3 | 1.6 | 1.8 | 2.1 |
| 4 years | 1.0 | 1.1 | 1.1 | 1.2 | 1.2 | 1.3 | 1.3 | 1.4 | 1.4 | 1.5 | 1.8 | 2.2 | 2.7 |
| 5 years | 1.1 | 1.1 | 1.2 | 1.2 | 1.3 | 1.3 | 1.4 | 1.5 | 1.6 | 1.6 | 2.1 | 2.7 | 3.5 |
| 6 years | 1.1 | 1.1 | 1.2 | 1.3 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 2.5 | 3.3 | 4.5 |
| 7 years | 1.1 | 1.2 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.8 | 1.9 | 2.0 | 2.9 | 4.1 | 5.8 |
| 8 years | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.8 | 1.9 | 2.1 | 2.2 | 3.3 | 5.0 | 7.4 |
| 9 years | 1.1 | 1.2 | 1.3 | 1.4 | 1.6 | 1.7 | 1.9 | 2.1 | 2.2 | 2.5 | 3.9 | 6.0 | 9.5 |
| 10 years | 1.1 | 1.2 | 1.3 | 1.5 | 1.6 | 1.8 | 2.0 | 2.2 | 2.5 | 2.7 | 4.5 | 7.4 | 12.2 |
| 15 years | 1.2 | 1.3 | 1.6 | 1.8 | 2.1 | 2.5 | 2.9 | 3.3 | 3.9 | 4.5 | 9.5 | 20.1 | 42.5 |
| 20 years | 1.2 | 1.5 | 1.8 | 2.2 | 2.7 | 3.3 | 4.1 | 5.0 | 6.0 | 7.4 | 20.1 | 54.6 | 148.4 |
| 25 years | 1.3 | 1.6 | 2.1 | 2.7 | 3.5 | 4.5 | 5.8 | 7.4 | 9.5 | 12.2 | 42.5 | 148.4 | 518.0 |
| 50 years | 1.6 | 2.7 | 4.5 | 7.4 | 12.2 | 20.1 | 33.1 | 54.6 | 90.0 | 148.4 | 1,808 | 22,026 | 268,337 |
| 75 years | 2.1 | 4.5 | 9.5 | 20.1 | 42.5 | 90.0 | 190.6 | 403.4 | 854.1 | 1,808 | 76,880 | $3,269,017$ | $139,002,156$ |
| 100 years | 2.7 | 7.4 | 20.1 | 54.6 | 148.4 | 403.4 | 1,097 | 2,981 | 8,103 | 22,026 | $3,269,017$ | $485,165,195$ | $72,004,899,337$ |

## Sudden or Continuously Accumulating Future Costs

The formulas presented above apply to costs or benefits at a particular time instant in the future. Now consider a cost, $c_{\text {rate }}$, which accumulates continuously as a cost per unit time. The present value of that cost is

$$
\begin{equation*}
c_{\text {present }}=\int_{t_{1}}^{t_{2}} c_{\text {rate }}(t) \cdot e^{-r \cdot t} d t \tag{5}
\end{equation*}
$$

where $t_{1}$ and $t_{2}$ are the future start and stop times of the cost. If the cost-rate is constant then $c_{\text {rate }}$ can be pulled out of the integral so the factor $\delta$ can be quantified:

$$
\begin{equation*}
\delta=\int_{t_{1}}^{t_{2}} e^{r \cdot t} d t=\frac{e^{r \cdot t_{2}}-e^{r \cdot t_{1}}}{r} \tag{6}
\end{equation*}
$$

## Rate Types

A challenging aspect of discounting is the determination of the interest rate, $r$. Four rates are relevant in this discussion:
$r_{n}=$ nominal interest rate for money saved or invested
$r_{i}=$ rate of inflation
$r_{r}=$ real interest rate
$r_{p}=$ pure time preference rate

What matters when invested money is compounded with $r_{n}$ is what is left after inflation has taken some of that profit. That is the reason Irving Fisher postulated that we use

$$
\begin{equation*}
r_{r}=r_{n}-r_{i} \tag{7}
\end{equation*}
$$

For the same reason it is appropriate to use $r_{r}$ in discounting, because inflation will eat away at money invested to pay for future costs.

## Canadian Rates

The variation in some Canadian rates is shown in Figure 1. The prime rate minus $2 \%$ is selected as the nominal rate because of data availability, and because the nominal bank rate is usually a couple of percentage points below prime. The black line in Figure 1 shows the variation in the real interest rate. The average real interest rate, calculated with the shown values, is precisely $1 \%$. The average in our millennium is near zero, and recently it has been negative.


Figure 1: Canadian rates.

## Assessing Future Costs Today

When we today estimate the future cost of, say, earthquake damage we use present-day construction cost tables. By the time the cost occurs it will have increased by inflation. Furthermore, as postulated by Fisher in Eq. (7), inflation will eat away at the profit of any money put aside for paying for the damage. This means the discount factor on the cost estimated today is

$$
\begin{equation*}
\delta=e^{\left(r_{n}-2 r_{i}\right) t} \tag{8}
\end{equation*}
$$

## Discounting Costs of Uncertain Occurrence Time

Consider a future event in which a utility value is realized. The typical example is a possible future structural failure that is associated with the cost $c_{f}$. When a mitigation action at the present time is contemplated it is appropriate to discount the future cost to present value. Otherwise the impact of the potential future loss is overvalued. It is conventional to employ an exponential discounting function so that the present value is

$$
\begin{equation*}
c_{p}=c_{f} \cdot \exp (-r \cdot t) \tag{9}
\end{equation*}
$$

where $r$ is the annual real interest rate, i.e., the actual interest rate minus inflation, and $t$ is the time, in years, from present to the time that the cost is incurred. Usually, both $r$ and $t$ are uncertain. The uncertainty in $r$ is modelled by a random variable. The uncertainty in $t$ is addressed by an occurrence model. If the Poisson occurrence model is employed then the time until the first occurrence is modelled by the exponential distribution, with probability density function

$$
\begin{equation*}
f(t)=\lambda \cdot \exp (-\lambda \cdot t) \tag{10}
\end{equation*}
$$

where $\lambda$ is the annual rate of occurrence of the Poisson process. If a long period of time is considered, i.e., approaching 100 years, then an approximation for the expected present value is

$$
\begin{align*}
E\left[c_{p}\right] & =\int_{0}^{\infty} c_{p} \cdot f(t) \cdot d t \\
& =\int_{0}^{\infty} c_{f} \cdot \exp (-r \cdot t) \cdot f(t) \cdot d t  \tag{11}\\
& =\int_{0}^{\infty} c_{f} \cdot \exp (-r \cdot t) \cdot \lambda \cdot \exp (-\lambda \cdot t) \cdot d t \\
& =c_{f} \cdot \frac{\lambda}{r+\lambda}
\end{align*}
$$

## Discounting Environmental Costs

When we expect to receive an invoice for a future cost, then it is financially rational to discount it before it is compared or added to present costs. However, some costs are not paid by invoice. Examples are environmental impacts and human injuries; should those
costs be discounted, i.e., reduced, before they are entered into the total lifecycle cost that is calculated today? If yes, which rate should be used? Many relevant discussions have been held in the context of cost of environmental damage. The Stern Review and the follow-up by Nordhaus are famous examples (Nordhaus 2007; Stern 2006). Several others have paid specific attention to the discounting problem (Kula and Evans 2011; Moxnes 2014). The problem has also been addressed in the structural safety community (Nishijima et al. 2007). An early and interesting viewpoint is that discounting because of capital growth is acceptable, but discounting because of pure time preference is not; it may also be acceptable to apply discounting if it is possible to express the damage in monetary units, if it is possible to compensate future generations for the damage, and if they would be satisfied with such compensation (Hellweg et al. 2003).

## References

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