

Code Calibration

Structural design is decision-making under uncertainty. Loads and capacities are invariably uncertain. However, current practice rarely employs explicit cost-benefit considerations to determine the design and, hence, the safety of the structure. Instead, the common practice is to set forth “design equations” in building codes and material standards with safety coefficients calibrated to past accepted practice. When a new code is in the making, a code committee often attempts to determine the reliability index implied by past practice, which forms the “target reliability” for future design equations. The act of determining the safety coefficients that meet the target reliability is called code calibration.

Design Equations Vs. Limit-state Functions

Consider the basic reliability problem, which is expressed in terms of the two random variables R and S , representing the capacity and demand, respectively, and the limit-state function

$$g = R - S \quad (1)$$

The *design equation* associated with that *limit-state function* is

$$\phi \cdot R_k = \gamma \cdot S_k \quad (2)$$

where ϕ =safety coefficient applied to the capacity, R_k =characteristic value of the capacity, γ =safety coefficient applied to the load, and S_k =characteristic value of the load. ϕ and γ are called “partial safety coefficients” because different safety coefficients are applied to capacities and loads. Typical values satisfy the inequalities $\phi < 1$ and $\gamma > 1$. The characteristic values of R and S could be the mean values of those random variables, but typically more conservative values are used. Specifically, the load-value that has a 5% chance of being exceeded and the capacity value that has a 5% chance of being subceeded are often used.

Code Calibration for the Basic Reliability Problem

In the context of the basic reliability problem, code calibration implies determining the values of ϕ and γ in Eq. (2) that yields the desired reliability index value. Professor Niels Lind presented a solution to this problem in the late 1960s. The first step in the derivation is to express the characteristic value of the capacity as

$$R_k = \mu_R - k_R \cdot \sigma_R = \mu_R - k_R \cdot \delta_R \cdot \mu_R = \mu_R \cdot (1 - k_R \cdot \delta_R) \quad (3)$$

and similarly, for the load:

$$S_k = \mu_S \cdot (1 + k_S \cdot \delta_S) \quad (4)$$

For the Normal distribution it turns out that $k_R = k_S = 1.64$. Next, two ratios are defined for future reference; namely, the “characteristic safety factor”

$$\lambda_k = \frac{R_k}{S_k} \quad (5)$$

and the “central safety factor”

$$\lambda_o = \frac{\mu_R}{\mu_S} \quad (6)$$

Having established those concepts, the reliability index for the basic reliability problem is calculated by the MVFOSM approach, assuming that R and S are independent:

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \quad (7)$$

To approach an analytical solution to this code calibration problem, Professor Lind made the following approximation:

$$\sigma_g = \sqrt{\sigma_R^2 + \sigma_S^2} \approx \alpha \cdot (\sigma_R + \sigma_S) \quad (8)$$

with $\alpha=0.75$, which is a fairly accurate assumption so long as the values of σ_R and σ_S do not differ too much. With that approximation, the reliability index is rewritten as

$$\beta = \frac{\mu_R - \mu_S}{\alpha \cdot (\sigma_R + \sigma_S)} \Rightarrow \mu_R - \mu_S = \beta \cdot \alpha \cdot (\sigma_R + \sigma_S) \quad (9)$$

Dividing through by μ_S yields

$$\frac{\mu_R}{\mu_S} - 1 = \beta \cdot \alpha \cdot \left(\frac{\sigma_R \cdot \mu_R}{\mu_S} + \frac{\sigma_S \cdot \mu_S}{\mu_S} \right) \Rightarrow \lambda_o - 1 = \beta \cdot \alpha \cdot (\delta_R \cdot \lambda_o + \delta_S) \quad (10)$$

Solving for the central safety factor yields

$$\lambda_o = \frac{1 + \beta \cdot \alpha \cdot \delta_S}{1 - \beta \cdot \alpha \cdot \delta_R} \quad (11)$$

Now consider the characteristic safety factor, where the central safety factor actually appears once Eqs. (3) and (4) have been substituted:

$$\lambda_k = \frac{\mu_R \cdot (1 - k_R \cdot \delta_R)}{\mu_S \cdot (1 + k_S \cdot \delta_S)} = \lambda_o \cdot \frac{(1 - k_R \cdot \delta_R)}{(1 + k_S \cdot \delta_S)} \quad (12)$$

Substitution of Eq. (11) yields

$$\lambda_k = \frac{(1 + \beta \cdot \alpha \cdot \delta_S)}{(1 - \beta \cdot \alpha \cdot \delta_R)} \cdot \frac{(1 - k_R \cdot \delta_R)}{(1 + k_S \cdot \delta_S)} \quad (13)$$

Finally, recognizing that $\lambda_k = R_k/S_k$, Eq. (13) is rearranged to match the format of Eq. (2):

$$\underbrace{\frac{(1 - \beta \cdot \alpha \cdot \delta_R)}{(1 - k_R \cdot \delta_R)}}_{\phi} \cdot R_k = \underbrace{\frac{(1 + \beta \cdot \alpha \cdot \delta_S)}{(1 + k_S \cdot \delta_S)}}_{\gamma} \cdot S_k \quad (14)$$

Eq. (14) is the sought link between the value of ϕ and γ in Eq. (2) and value of the reliability index, β .

Geometry Link

The fundamental problem in code calibration is to link the design equation, such as the one in Eq. (2) with the limit-state function, such as the one in Eq. (1). Analytical solutions to that challenge may not exist. However, in certain situations there may exist deterministic geometry parameters that appear in both equations, facilitating an analytical combination. As a conceptual example, consider a problem with two loads and one capacity, e.g., a simply supported beam subjected to dead load and live load. The design equation employed by the engineer to check the bending moment capacity is

$$\phi \cdot M_{Capacity} = \gamma_{Dead} M_{Dead} + \gamma_{Live} M_{Live} \quad (15)$$

Assuming the bending moment capacity can be calculated from a yield stress, f_y , and that the loads, q_{Dead} and q_{Live} , are uniformly distributed, Eq. (15) is rewritten as

$$\phi \cdot \left(f_{y,k} \cdot \frac{I}{z} \right) = \gamma_{Dead} \left(\frac{q_{Dead,k} \cdot L^2}{8} \right) + \gamma_{Live} \left(\frac{q_{Live,k} \cdot L^2}{8} \right) \quad (16)$$

The corresponding limit-state function is

$$g = f_y \cdot \frac{I}{z} - \frac{q_{Dead} \cdot L^2}{8} - \frac{q_{Live} \cdot L^2}{8} \quad (17)$$

where f_y , q_{Dead} , and q_{Live} are random variables. Solving Eq. (16) for the geometry parameter I and substituting it into Eq. (17) yields

$$g = \frac{f_y}{\phi \cdot f_{y,k}} \cdot \left(\gamma_{Dead} \left(\frac{q_{Dead,k} \cdot L^2}{8} \right) + \gamma_{Live} \left(\frac{q_{Live,k} \cdot L^2}{8} \right) \right) - \frac{q_{Dead} \cdot L^2}{8} - \frac{q_{Live} \cdot L^2}{8} \quad (18)$$

Terms without any random variable can be cancelled without changing the failure probability; hence, the limit-state function is simplified to

$$g = \frac{f_y}{\phi \cdot f_{y,k}} \cdot (\gamma_{Dead} \cdot q_{Dead,k} + \gamma_{Live} \cdot q_{Live,k}) - q_{Dead} - q_{Live} \quad (19)$$

Eq. (19) is a limit-state function that contains the safety coefficients of the design equation. As a result, it can be used directly for code calibration, following this procedure:

1. Obtain probabilistic information about the random variables
2. Set a target reliability
3. Iterate:
 - a. Select trial values for the safety coefficients
 - b. Carry out the reliability analysis with the limit-state function in Eq. (19)
 - c. Compare the reliability index to the target reliability

- d. Try different values for the safety coefficients, because there is no unique solution

Real-world Code Calibration

Yet to be written.