Bayesian Network Models

A Bayesian network is essentially a collection of discrete random variables. As an introduction it may be useful think of each random variable as a component, or "node," in a physical network, such as a bridge in a road network. The realizations of each random variable represent possible states of the corresponding component, such as "open" and "closed." Importantly, the probability mass function (PMF) of most of the random variables in a Bayesian network is dependent on the realization of "parent" random variables in the network. As a result, a central part of Bayesian network modelling is to establish conditional PMFs of the form $p(x_i|x_j,x_k,\ldots)$, where X_i is the "child node" and X_j, $X_k,\ldots$ are "parent nodes." As an example of this dependence structure, consider the Bayesian network in Figure 1. This representation is called a "directed acyclic graph" (DAG), where each random variable, i.e., node, is drawn as a circle. Arrows are drawn from parent nodes to child nodes to visualize the dependence structure. In this particular example it is seen that the nodes X1 and X_2 do not have parent nodes, while X_3 is a child node that has both X_1 and X_2 as parent nodes. Therefore, the PMF for X_3 is of the form $p(x_3|x_1,x_2)$. Figure 1 also shows that X_4 has X_1 as parent node, and X_5 has X_4 as parent node. As a result, the joint PMF for all the random variables is, according to the multiplication rule of probability:

$$p(x_1,x_2,x_3,x_4,x_5) = p(x_5 \mid x_4)p(x_4 \mid x_1)p(x_3 \mid x_1,x_2)p(x_1)p(x_2) \tag{1}$$

This result is important because a Bayesian network is essentially a tool to establish the joint probability distribution for a collection of random variables. Once this result is obtained, the probability of any system state can be computed by summation of probabilities, i.e., summation of the right-hand side in Eq. (1) in accordance with the theorem of total probability. In fact, the use of conditional PMFs visualized in Figure 1 has reduced the number of probability values from m^{5-1} in the left-hand side of Eq. (1) to $m^2+m^2+m^3+m+m-5$, i.e., far less, in the right-hand side of Eq. (1), where m is the number of possible outcomes of each random variables. The Bayesian network formulation also makes it straightforward to incorporate new information about individual components. For example, if it becomes known that a bridge is closed then the uncertainty in that node is removed and replaced by the known outcome. In this way, the network model is readily updated with real-time information, for example in the aftermath of an earthquake.

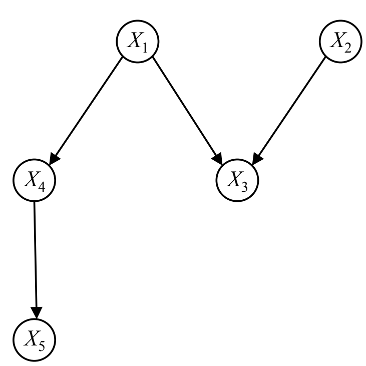

Figure 1: Bayesian network.