Analysis of Determinate Structures

For statically determinate structures it is possible to determine the internal forces, such as bending moments and shear forces, by equilibrium alone. That means the stiffness of the structure does not influence the distribution of forces in the structure. It also means that a statically determinate structure will never experience internal forces due to temperature changes and settlements. The most common internal force diagrams are:

- Bending moment diagram (BMD)
- Shear force diagram (SFD)
- Axial force diagram (AFD)

Such diagrams, shown in Figure 1, display the value of the bending moment, shear force, or axial force along a structural member. The conventions used in the notes posted on this website are shown in Figure 1.

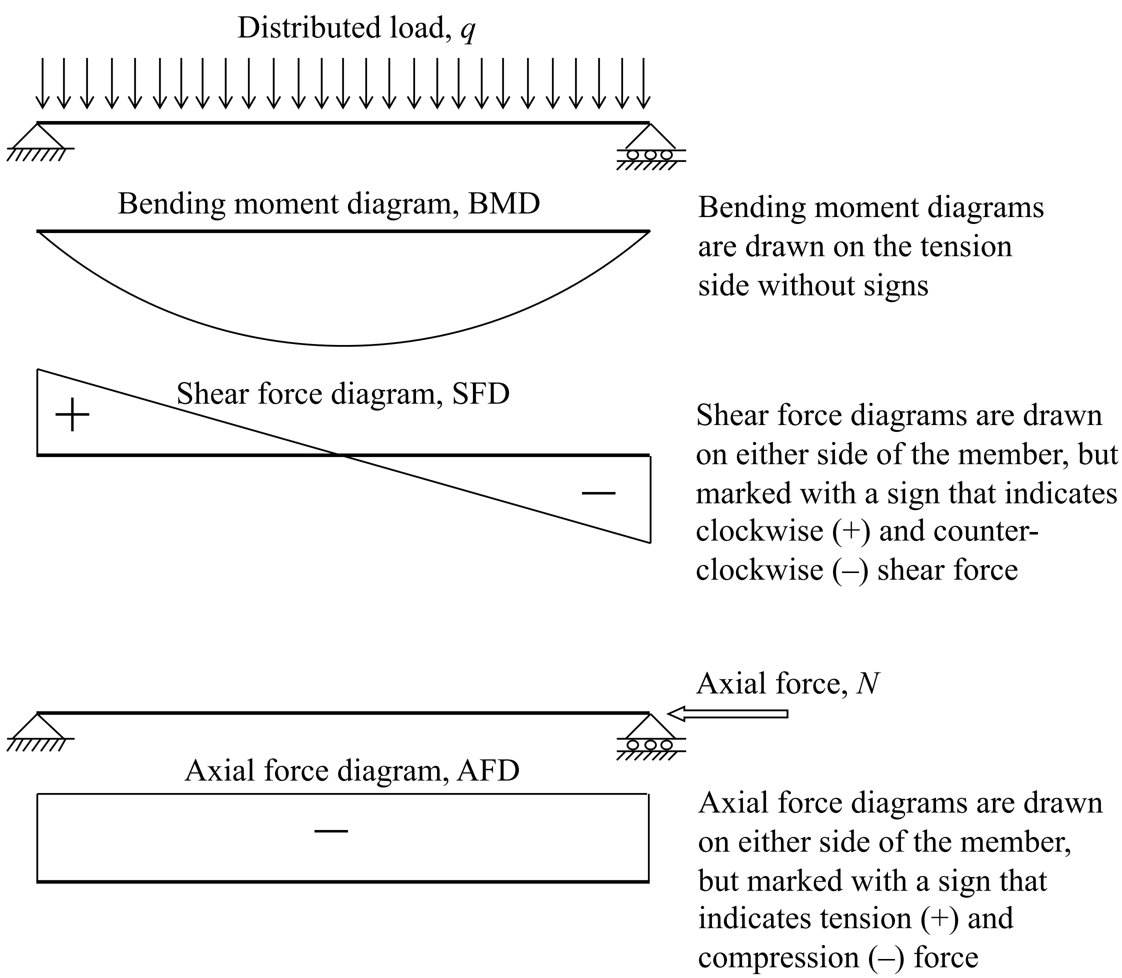

Figure 1: Conventions for bending moment, shear force, and axial force diagrams.

Trusses

A truss consists of members that can carry axial force only. If the truss structure is statically determinate it is possible to determine the axial force in every member by equilibrium considerations. A procedure to carry out the analysis for 2D truss structures is:

1. Determine the reaction force(s) at every support by global equilibrium equations.
2. Starting at a joint with maximum two unknown axial forces, determine the axial forces by equilibrium. Do this by drawing a force polygon of the axial forces meeting in that joint. Equilibrium requires the force polygon to be closed. The direction of the force-arrows determines whether the member is in tension or compression. An arrow towards the joint indicates a member in compression because the equal and opposite force pushes towards the member.
3. Move from joint to joint, always moving to a joint with maximum two unknown axial forces.

Frames

Frames have members that carry load by bending moment, shear force, and axial force. For certain statically determinate problems it is possible to determine the section force diagrams simply by recognizing basic beam cases in the structure. In particular, for structures that have cantilevers and simply supported beams the section force diagrams for those parts are readily known from the basic beam cases. In general, for statically determinate 2D frames the following procedure is suggested:

1. Determine the reaction force(s) at every support by global equilibrium equations.
2. Determine the bending moment at selected locations: typically, member ends and midpoints. Do this by imagining that the member is cut and compute the bending moment that must act on either side of the cut to maintain equilibrium. Sketch the shape of the bending moment diagram by connecting the end points of those lines. The polynomial order of the diagram depends on the load that the member is subjected to; see Table 1 details, which are obtained by solving the differential equation for beams.
3. On another drawing of the structure, sketch the shear force diagram. As shown in Table 1 the polynomial order of the shear force diagram is one less than that of the bending moment diagram. To understand whether the shear force is clockwise or counterclockwise, "walk" along the bending moment diagram from left to right. A downhill walk implies a positive, i.e., clockwise shear force. An uphill walk implies a counter-clockwise shear force. The value of the shear force is obtained from the equilibrium equation $V=dM/dx=\Delta M/\Delta x$ when the bending moment diagram is linear, or more generally by cutting the member and computing the shear force that must act on either side of the cut.
4. The axial force in each member is computed equilibrium at the joints. The sum of the forces in each joint, from shear forces and axial forces, must equal zero.

In regards to the above procedures, alternative approaches and habits exist. Please attend my lectures in CIVL 332 for additional comments and examples. Some engineers prefer to establish the shear force diagram before the bending moment diagram. In particular, inspired by the information in Table 1, some prefer to establish the shear force diagram by integration of the load, and the bending moment by integration of the shear force. This is rather straightforward for horizontal beams, but cumbersome for general frame structures. For that reason, the general procedure given above is preferred.

Table 1: Polynomial order of section force diagrams.

Load	**No distributed load**	**Uniform**	**Linear**
Shear force	Constant	Linear	2nd order
Bending moment	Linear	2nd order	3rd order
Rotation	2nd order	3rd order	4th order
Displacement	3rd order	4th order	5th order