Influence Lines

An influence line is drawn along a member in the same way as for example a bending moment diagram. However, influence lines mean something quite different than a section force diagram. Specifically, an influence line shows the variation of a section force—or a support reaction—at a particular location due to a load that travels along the member. The value of the influence diagram at a particular location shows the value of the selected section force when the load is in that position.

In this document, emphasis is placed on our ability to approximately sketch the shape of the influence line. This is because a key use of influence lines is to determine the worst-case positioning of loads. It turns out that only a rough assessment of the shape of the influence is necessary for this purpose. A principle named after Muller and Breslau is utilized to establish the shape of influence lines.

The nature of influence lines is different for statically determinate and indeterminate structures. For determinate structures the influence lines are linear, while for indeterminate structures they are curved. Regardless, the Muller-Breslau principle is the same, with the following procedure:

1. Select the location where the reaction or internal force should be monitored
2. Remove the capacity of the structure to carry that particular reaction or internal force
3. Introduce a displacement or rotation that corresponds to the removed restraint
4. The resulting displaced shape is the shape of the influence line

This procedure provides the shape of the influence line for a concentrated load travelling over the structure. Conversely, the value of a reaction or internal force due to a uniform unit load acting over a region is equal to the area under the influence line in that region. That fact is substantiated as follows: Consider an influence line with ordinates $\eta(x)$. Next, consider the uniform load to be a collection of concentrated loads:

$$dP = q_z \cdot dx \tag{1}$$

According to the definition of what an influence line is, the internal force value, dF, due to the concentrated load is:

$$dF = dP \cdot \eta(x) = q_z \cdot dx \cdot \eta(x) \tag{2}$$

Next, integrate to obtain the total reaction or internal force due to a distributed load between points A and B:

$$F = \int_A^B dF = \int_A^B q_z \cdot \eta(x) \cdot dx = q_z \cdot \underbrace{\int_A^B \eta(x) \cdot dx}_{\text{Area of inf. diagram}} \tag{3}$$

The sign convention of influence diagrams is as follows: Where the traveling load would push the influence line towards the structure, the load will produce a positive reaction or internal force. Where the traveling load would push the influence line further away from

the structure, the load will produce a negative reaction or internal force. This understanding allows the worst-case positioning of loads. For example, to get the largest possible positive reaction at a location, loads should be placed wherever they would push the influence diagram towards the structure.